第三屆的時頻分析與地球科學研討會

經驗模態分解法混波問題探討與解決方案

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Outline

- IMFs
- EMD
- Mode Mixing Problem
- Iterative Gaussian Filter
- Conclusion

EMD! Hot!

Google 學術搜尋 empirical mode decomposition 搜尋 進階學術搜尋

搜尋所有網站 搜尋所有中文網頁 搜尋繁體中文網頁

學術搜尋 任何時間 只包含書目/引用資料 建立電子郵件快訊 共約有204,000項查詢結果，這是第1-10項 。 (0.22 秒)

提示： 如只要搜尋中文（繁體）的結果，可使用學術搜尋偏好指定搜尋語言。

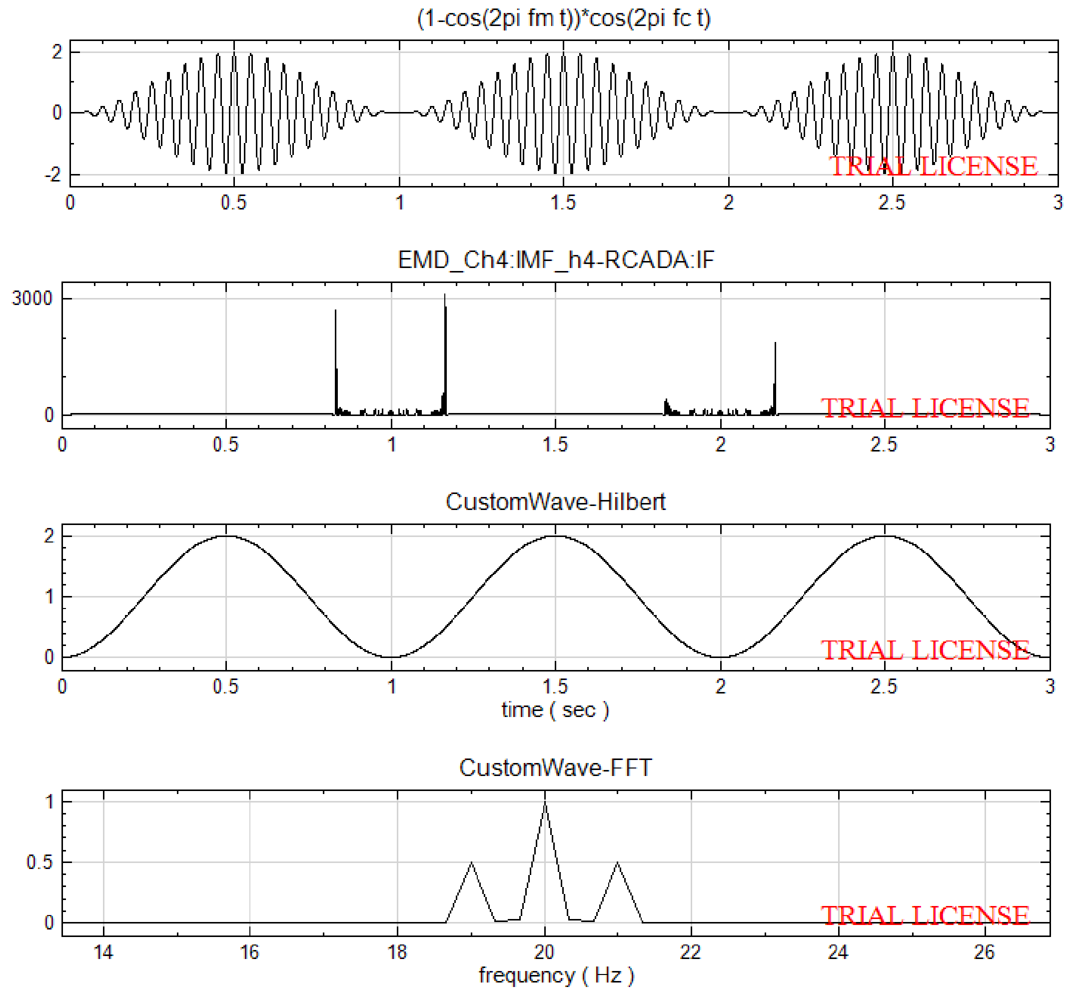
The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis — psu.edu [PDF]
NE Huang, Z Shen, SR Long, MC Wu, HH Shih, ... - Proceedings: ..., 1998 - JSTOR
Page 1. The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis ... 911 4. Intrinsic mode functions 915 5. The empirical mode decomposition method: the sifting process 917 6. Completeness and orthogonality 923 ...
被引用 3128 次 - 相關文章 - findit@ntnu - Other Service at NTNU - 全部共 27 個版本

[PDF] Empirical mode decomposition as a filter bank — psu.edu [PDF], NTNU館藏
P Flandrin, G Rilling, P Goncalves - IEEE signal processing letters, 2004 - Citeseer
Abstract— Empirical Mode Decomposition (EMD) has recently been pioneered by NE Huang et al. for adaptively representing nonstationary signals as sums of zero-mean AM-FM components [2]. In order to better understand the way EMD behaves in stochastic situations involving ...
被引用 440 次 - 相關文章 - HTML 版 - Fulltext@NTNU - NTNU館藏 - 全部共 8 個版本

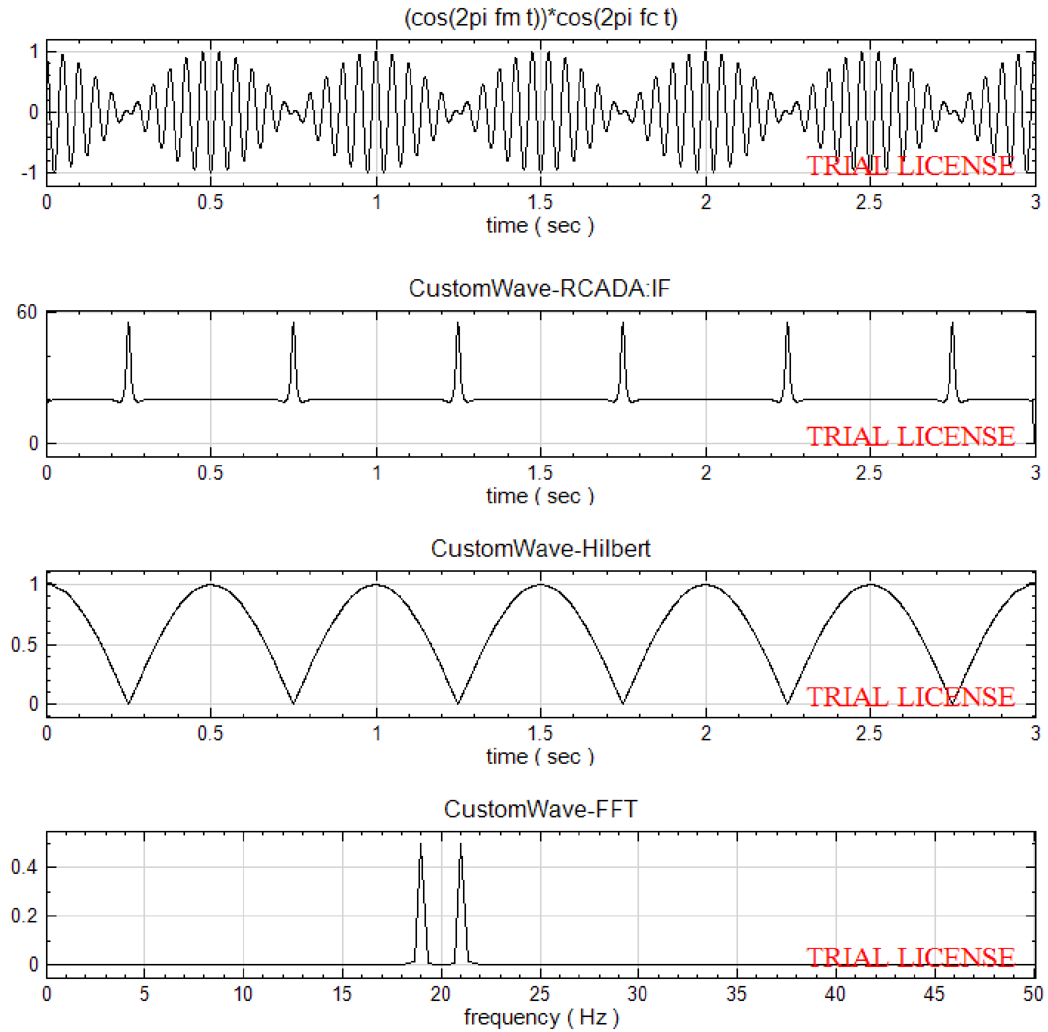
[PDF] On empirical mode decomposition and its algorithms — ens-lyon.fr [PDF]
G Rilling, P Flandrin, P Gonçalvès - IEEE-EURASIP workshop on ..., 2003 - perso.ens-lyon.fr
A new nonlinear technique, referred to as Empirical Mode Decomposition (EMD), has recently been pio- neered by NE Huang et al. for adaptively represent- ing nonstationary signals as sums of zero-mean AM- FM components [2]. Although it often proved re- markably effective [1, ...
被引用 282 次 - 相關文章 - HTML 版 - 全部共 7 個版本

Citation Number: 3128

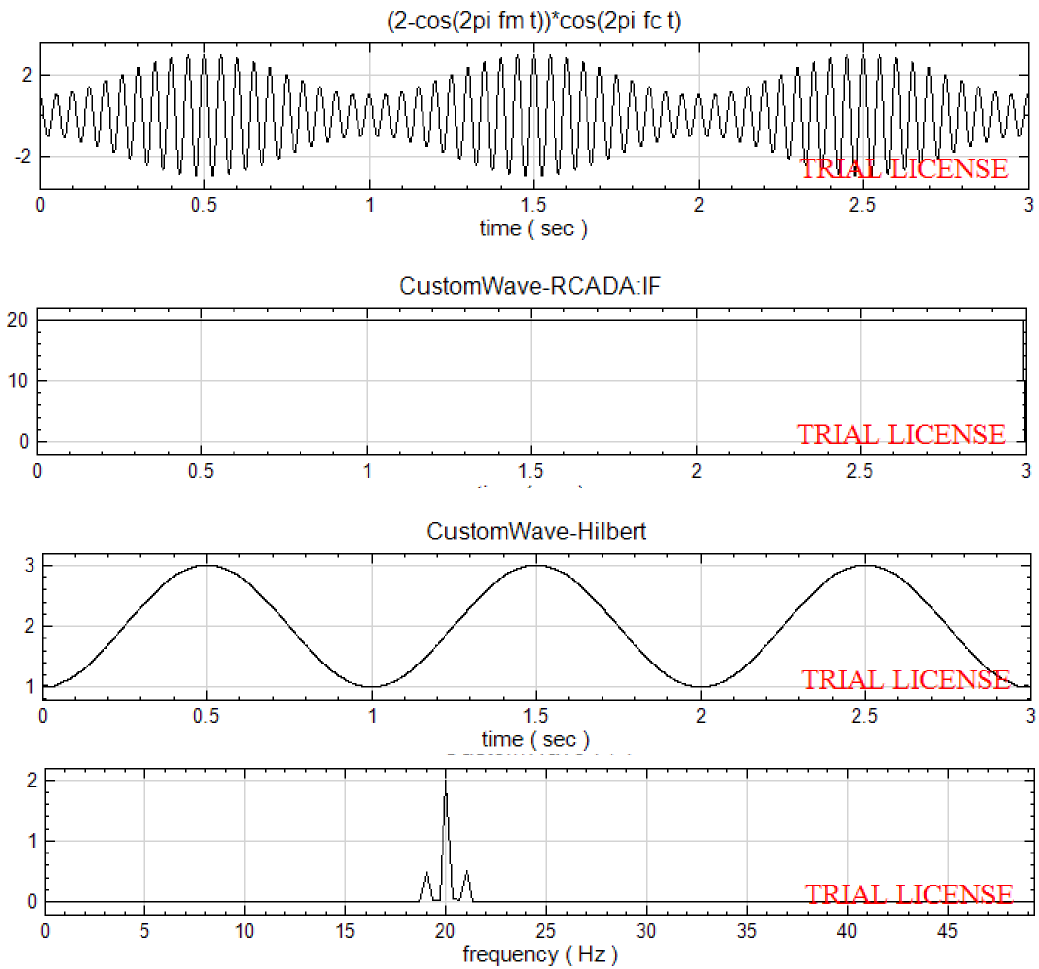
FFT v.s. Hilbert Transform



FFT v.s. Hilbert Transform

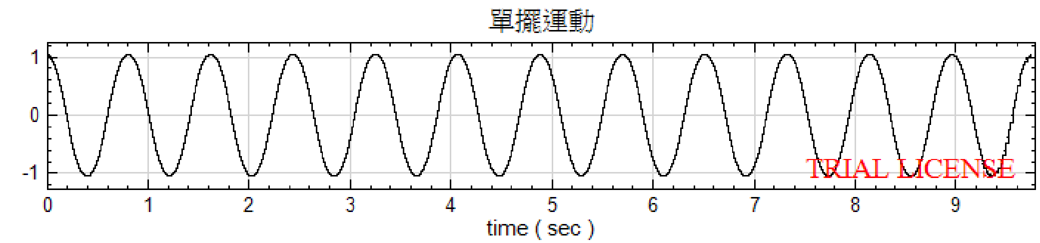


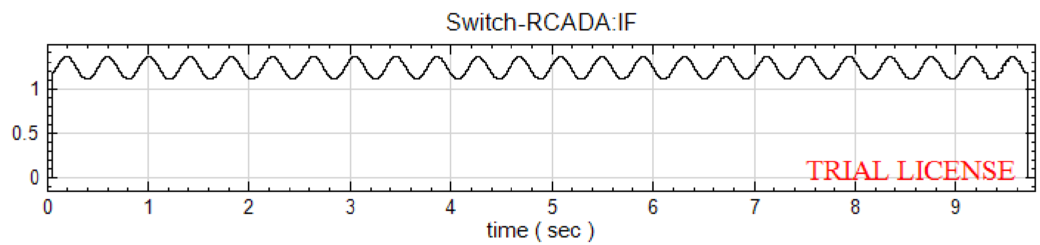
FFT v.s. Hilbert Transform



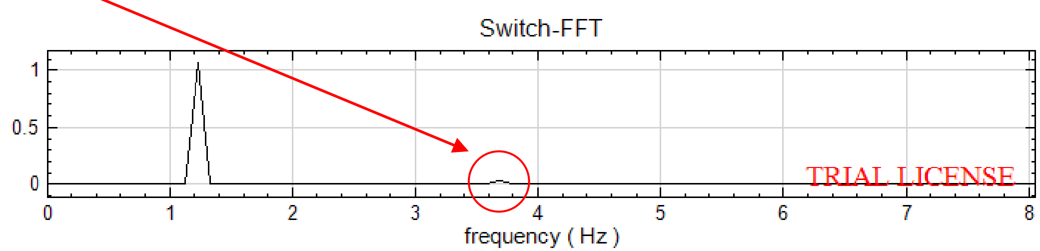
FFT v.s. Hilbert Transform

- $\ddot{\theta} + {}^{l}/_{g}\, sin\theta = 0$,



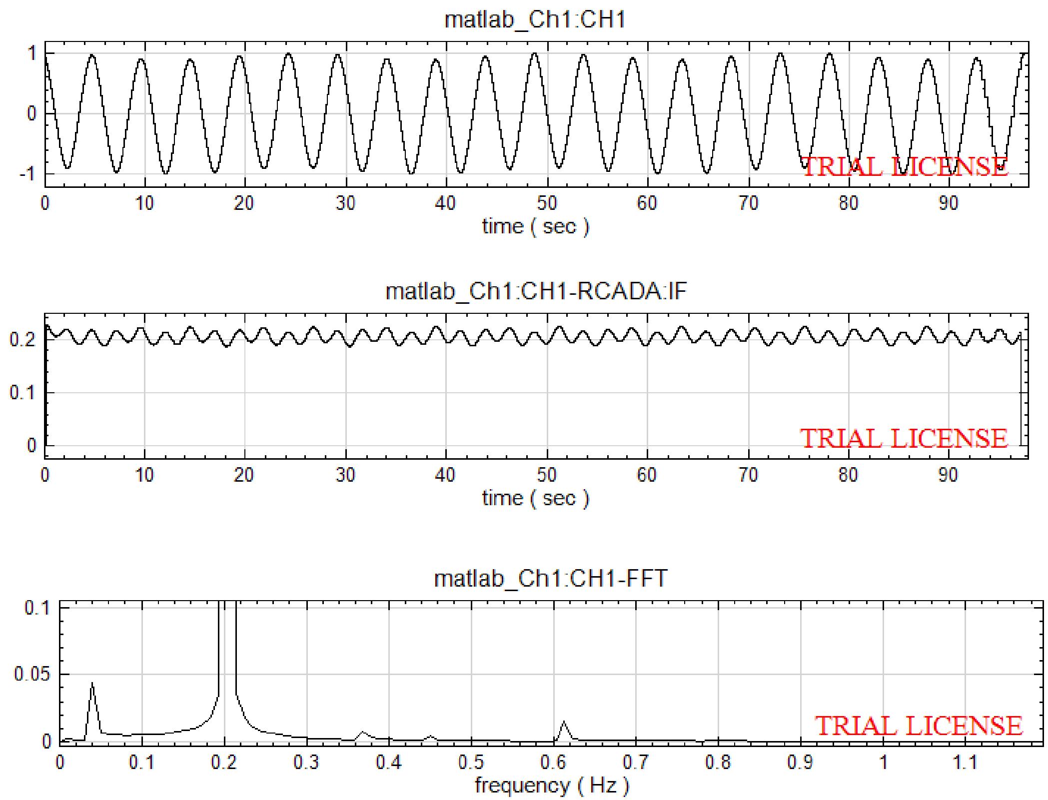


Third Harmonics

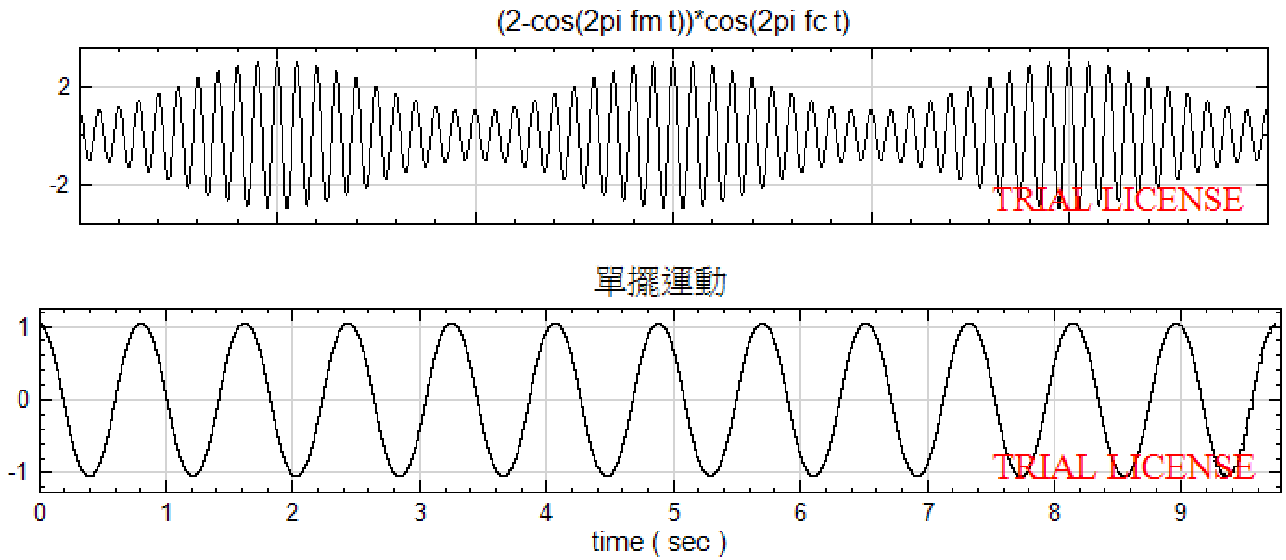


FFT v.s. Hilbert Transform

- $\frac{d^2x}{dt^2} + x + \epsilon x^3 = \gamma\cos(\omega t)$ ϵ=1,γ=0.1,ω=0.04



- Mono component signal or multi component signal?



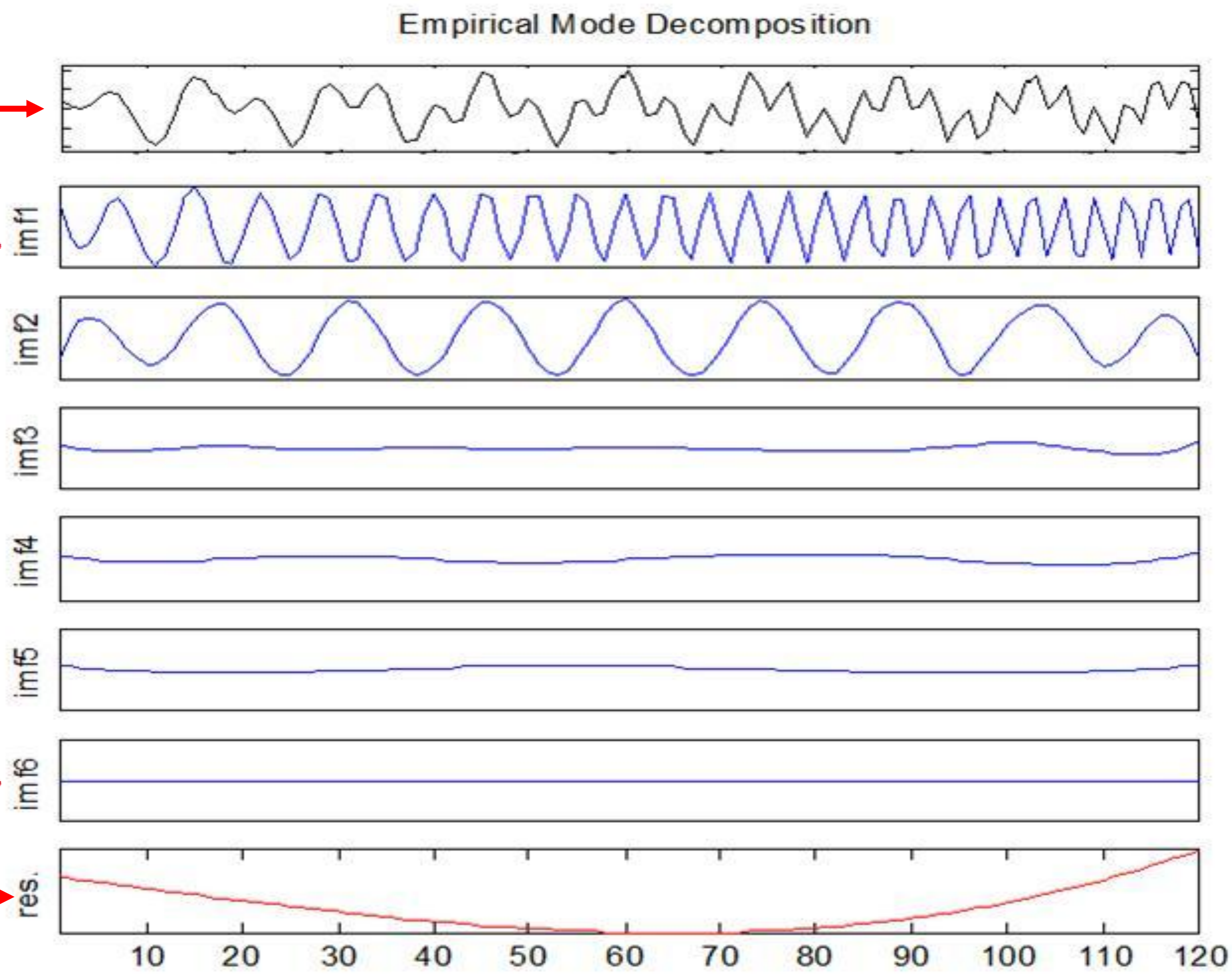
- EMD prefer to interpret the above signals as mono-component signals
  - Positive amplitude and positive IF.

Signal Decomposition

original signal

IMFs

residue



Ref [1]

Objectives of EMD

- EMD decomposes an arbitrary signal into several **intrinsic mode functions (IMFs)**:

$$S(t) = \sum_{k=1}^{K} S_k(t)$$

$$S_k(t) = A_k(t)\cos(\varphi_k(t))$$

- IMF can be considered as a **mono-component signal.**

What is IMF?

- An intrinsic mode function (IMF) is a function that satisfies two conditions: [Huang 1998]
  - (1) in the whole data set, the number of extrema and the number of zero crossings must either equal or differ at most by one; (Guaranteed the IMFs are narrow band signals)
  - (2) at any point, the mean value of the envelope defined by the local maxima and the envelope defined by the local minima is zero.

What is IMF?

- Zhihua Yang and Lihua Yang (2009) proves that Condition 1 of the IMF can really be deduced from Condition 2 and then gives the new definition of IMF:
  - An Intrinsic Mode Function (IMF) is a function that satisfies the condition that at any time instant, the mean value of the upper envelope as defined by the local maxima and the lower envelope as defined by the local minima is zero.

What is IMF?

- Robert C. Sharpley and Vesselin Vatchev(2006)
  - ***A function ψ is an IMF if and only if it is a weak-IMF whose spline envelopes satisfy the condition that the absolute value of the lower spline envelope is equal to the upper envelope and this common envelope is a quadratic polynomial. Furthermore, the common spline envelope is constant (i.e., ψ is an FM signal) if and only if Q = 1/P for the associated self-adjoint differential equation:***

$$\frac{d}{dt}\left(P(t)\frac{df(t)}{dt}\right)+Q(t)f(t)=0$$

What is IMF?

- El Hadji S. Diop, R. Alexandre, and A. O. Boudraa [2010]
  - An IMF is the solution of the following parabolic PDE

$$\begin{cases} \dfrac{\partial h}{\partial t} + \dfrac{h}{\delta^2} + \dfrac{1}{2}\dfrac{\partial^2 h}{\partial x} = 0 \\ h(x,0) = S(x) \end{cases}$$

What is IMF?

- Ingrid Daubechies, Jianfeng Lu1, Hau-Tieng Wu (2010)
  - A continuous function $f : R \to \mathbb{C}$, $f \in L^{\infty}(\mathbb{C})$ is said to be intrinsic-mode-type (IMT) with accuracy $\varepsilon > 0$ if $f = A(t)e^{j\phi(t)}$ with A and ϕ having the following properties:

$$A(t) \in C^1, \phi(t) \in C^2$$

$$\inf \phi'(t) > 0, \sup \phi'(t) < \infty$$

$$|A'(t)| \le \varepsilon |\phi'(t)|$$

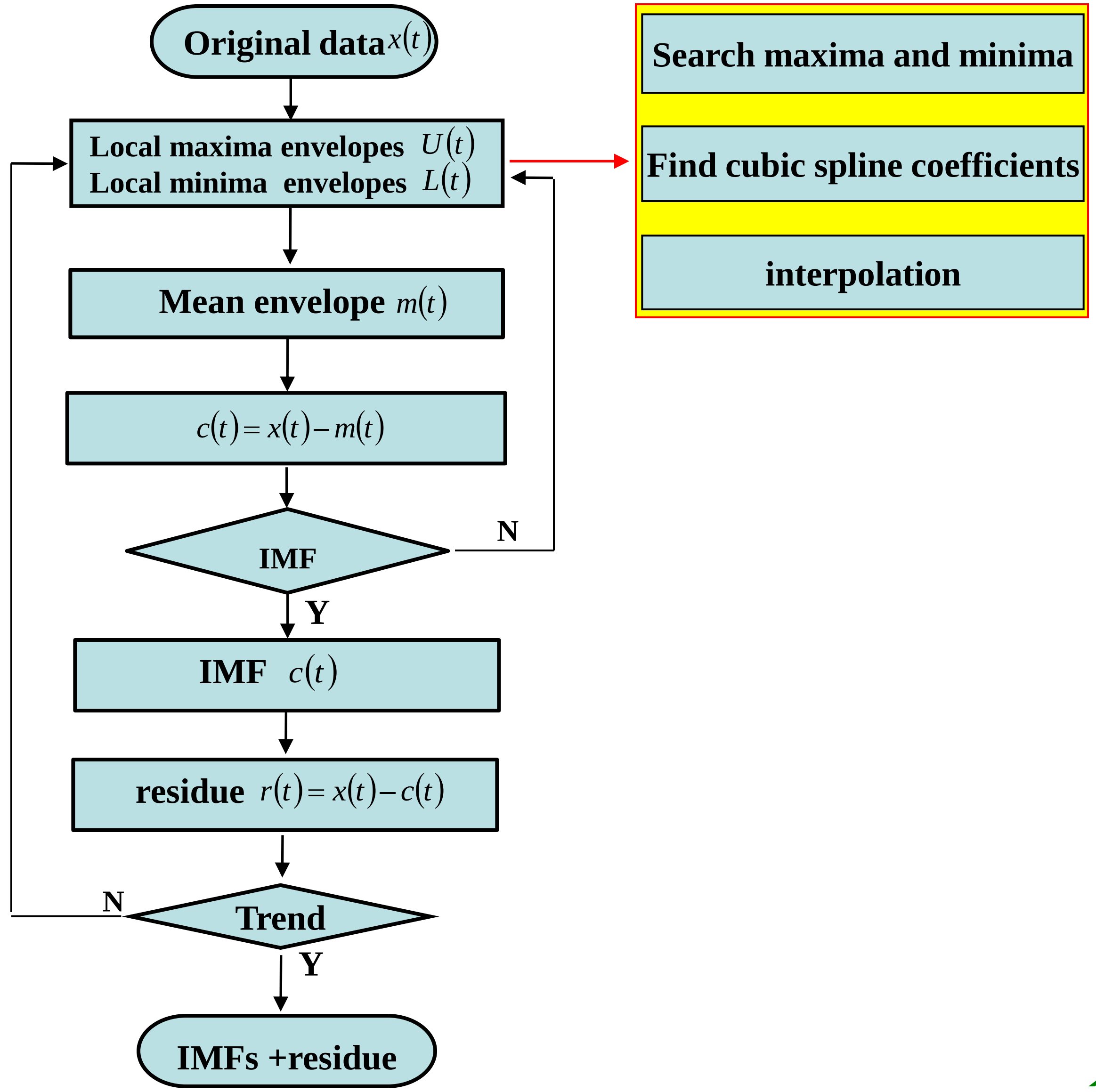
$$|\phi''(t)| \le \varepsilon |\phi'(t)|$$

How to obtain IMFs?

- Sifting Process (**Cubic Spline interpolation)** [Huang 1998]
- emd.ppt
- Some issues:
  - Location of Extrema
  - Envelops
  - Stop criterion
  - Boundary Extension

How to obtain IMFs?

- **Flow chart [7]**



Modified the Location of Extrema

- Z. Xu, B. Huang and S. Xu [2008]

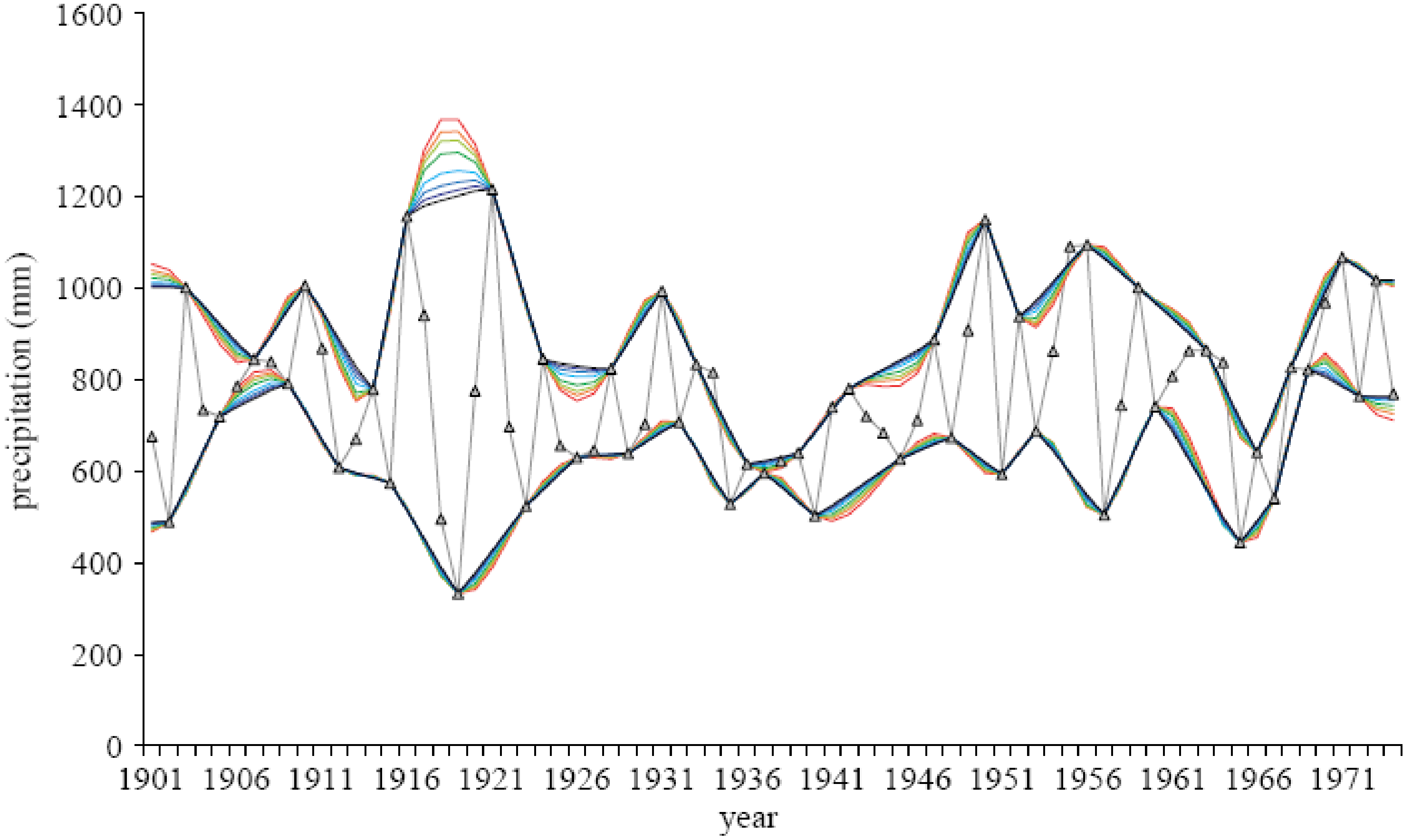
- Relocation Extrema by using parabolic interpolation
  - R.T. Rato, M.D. Ortigueira, A.G. Batista [2008]

Averaging Upper and Lower Envelopes

- B-spline:
  - Q. Chen , N. Huang [2006]
- High Order spline
  - Yannis Kopsinis and SteveMcLaughlin [2008]
- Rational Spline
  - Peel, M.C. , G.G.S. Pegram and T.A. McMahon [2008][2009]
- parabolic parameter spline interpolation algorithm
  - S.R. Qina,, Y.M. Zhong [2006]
- Higher order extrema
  - Li Lin , Ji Hongbing [2009]
- Piecewise parabolic polynomial
  - Zhengguang Xu , Benxiong Huang, Kewei Li [2010]
- Raised Cosine interpolation
  - A. Roy and J.F. Doherty [2010]
- Trigonometric interpolation
  - S. D. Hawley, L. E. Atlas,and H. J. Chizeck [2010]

Averaging Upper and Lower Envelopes

- Rational Spline
  - Peel, M.C. , G.G.S. Pegram and T.A. McMahon [2008]



Comuputing Mean Envelope Directly

- Constrained Optimization (quadratic programming)
  - Sylvain Meignen and Valérie Perrier [2007]
- Piecewise linear
  - M. G. Frei and I. Osorio [2007]
- Interpolate mean envelope by cubic spline using some special knots
  - Z. Xu, B. Huang and F. Zhang [2009]
- Local Integral Mean
  - Hong Hong, Xinlong Wang, and Zhiyong Tao [2009]
- PDE approaches
  - Eric Deléchelle, Jacques Lemoine, and Oumar Niang[2005]
  - El Hadji S. Diop, R. Alexandre, and A. O. Boudraa [2010]
- Window Averaging (Blackman Window)
  - F. Bao , X. Wanga, Z. Tao , Q. Wang S. Du [2010]

Stop Criterion

- Standard Deviation
  - Haung [1998]
- S Number
  - Haung [2003]
- Three Threshold Value
  - G. Rilling, P. Flandrin [2003]
- Energy Difference Tracking
  - Cheng Junsheng, Yu Dejie, Yang Yu [2006]
- Bandwidth
  - B. Xuan, Q. Xie and S. Peng [2007]

Stop Criterion Based on Bandwidth

$$\mathrm{H}[x(t)] = \frac{1}{\pi}\int \frac{x(\tau)}{t-\tau}\,d\tau$$

$$A[x(t)] = x(t) + jH[x(t)] = a(t)\exp(-j\phi(t))$$

$$B^2 = B_a^2 + B_f^2$$

$$B_a^2 = \int \left(\frac{a'(t)}{a(t)}\right)^2 a^2(t)\,dt = \int (B_t)^2 a^2(t)\,dt$$

$$B_f^2 = \int (\varphi'(t) - \langle\omega\rangle)^2 a^2(t)\,dt.$$

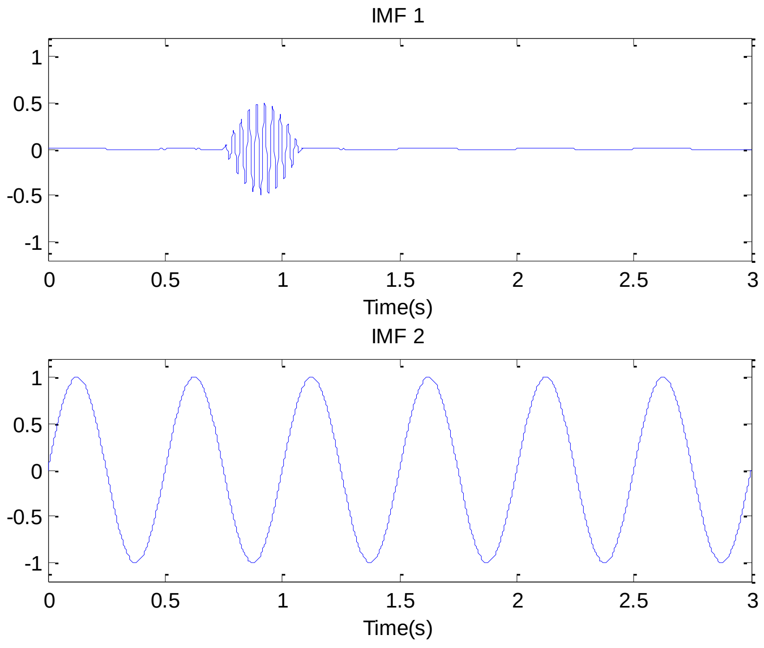
Boundary Extension

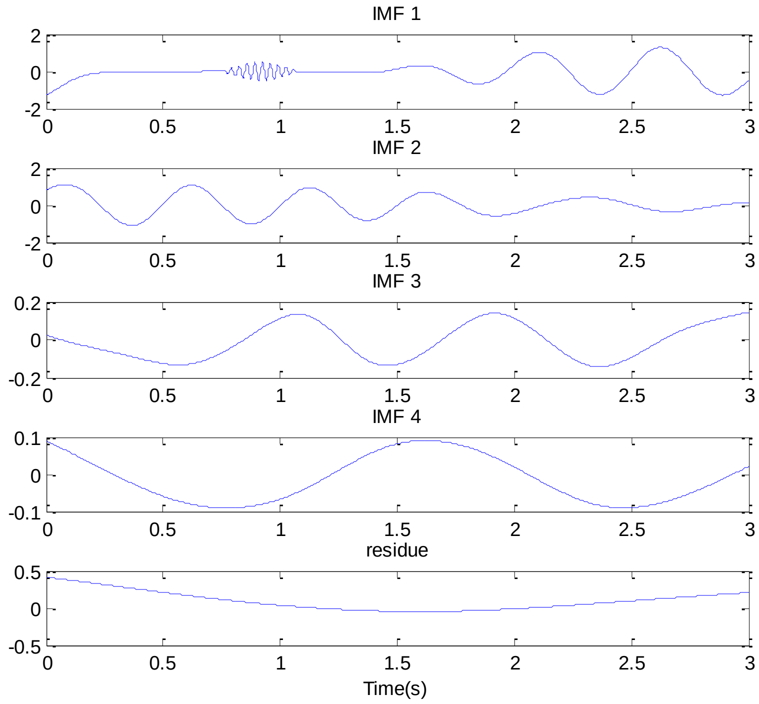
- characteristic wave extending method
  - Huang [1998]
- neural network extending
  - Y. Deng, W. Wang [2001]
- Mirror Extending
  - J. Zhao and D. Huang [2001]
- Data Extending
  - Zeng and He [2005]
- Similarity Search
  - J. Wang, Y. Peng and X. Peng [2007]
- Combinig End Mirror and least square polynomial extending
  - Zhang Qingjie, Zhu Huayong, Shen Lincheng[2010]
- Ratio extension (Be able to decompose Chirp Signal)
  - QIN WU and SHERMAN RIEMENSCHNEIDER [2010]

Mode Mixing

- [Huang 2009] Mode mixing which is defined as
  - a single Intrinsic Mode Function (IMF) consisting of signals of widely disparate scales,
  - a signal of a similar scale residing in different IMF components.
- Mode mixing is caused by
  - Signal intermittency [Huang 2009]
  - Limitation sifting process

Example

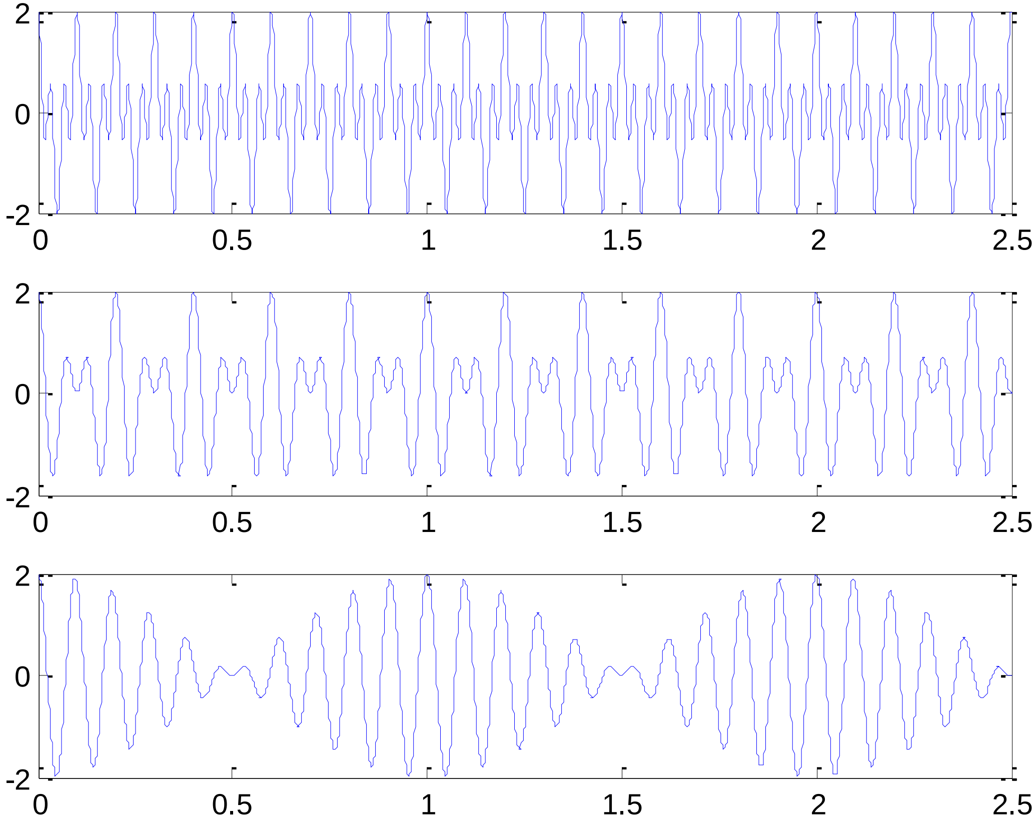

IMF 1
1
0.5
0
-0.5
-1
0
0.5
1
1.5
2
2.5
3
Time(s)
IMF 2
Time(s)


IMF 1
IMF 2
IMF 3
IMF 4
residue
Time(s)

Solution of Signal Intermittency

- Noise Assisted (Ensemble EMD)
  - ZHAOHUA WU and NORDEN E. HUANG [2009]
- COMPLEMENTARY ENSEMBLE EMD
  - JIA-RONG YEH, JIANN-SHING SHIEH and NORDEN E. HUANG [2010]
- Wavelet with Translation Invariance
  - Qin Pinle, Lin Yan, and Chen Ming [2008]
- Intermittency Test Algorithm
  - Build in Function of Visual Signal (developed by AnCad)
- Iterative Gaussian Window Averaging

One or two frequencies?

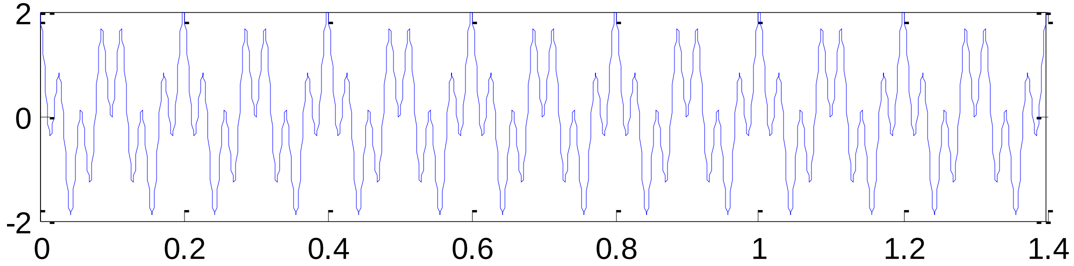


$$\cos(20\pi t)+\cos(60\pi t)$$

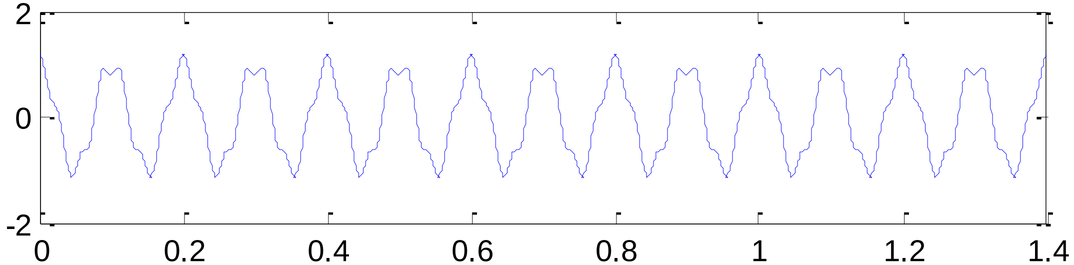
$$\cos(20\pi t)+\cos(30\pi t)$$

$$\cos(20\pi t)+\cos(22\pi t)$$

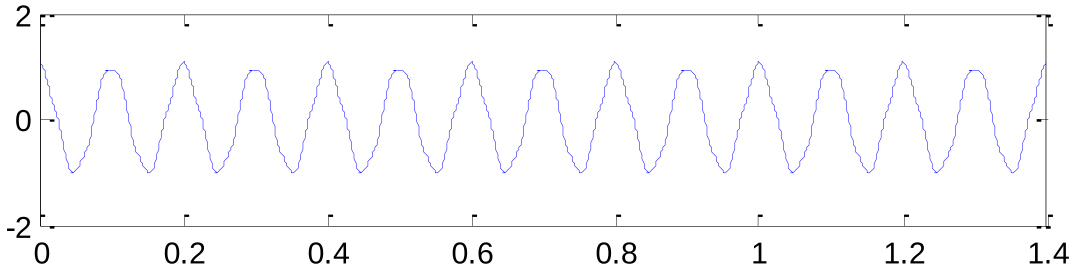
One or two frequencies?



$\cos(20\pi t)+\cos(70\pi t)$



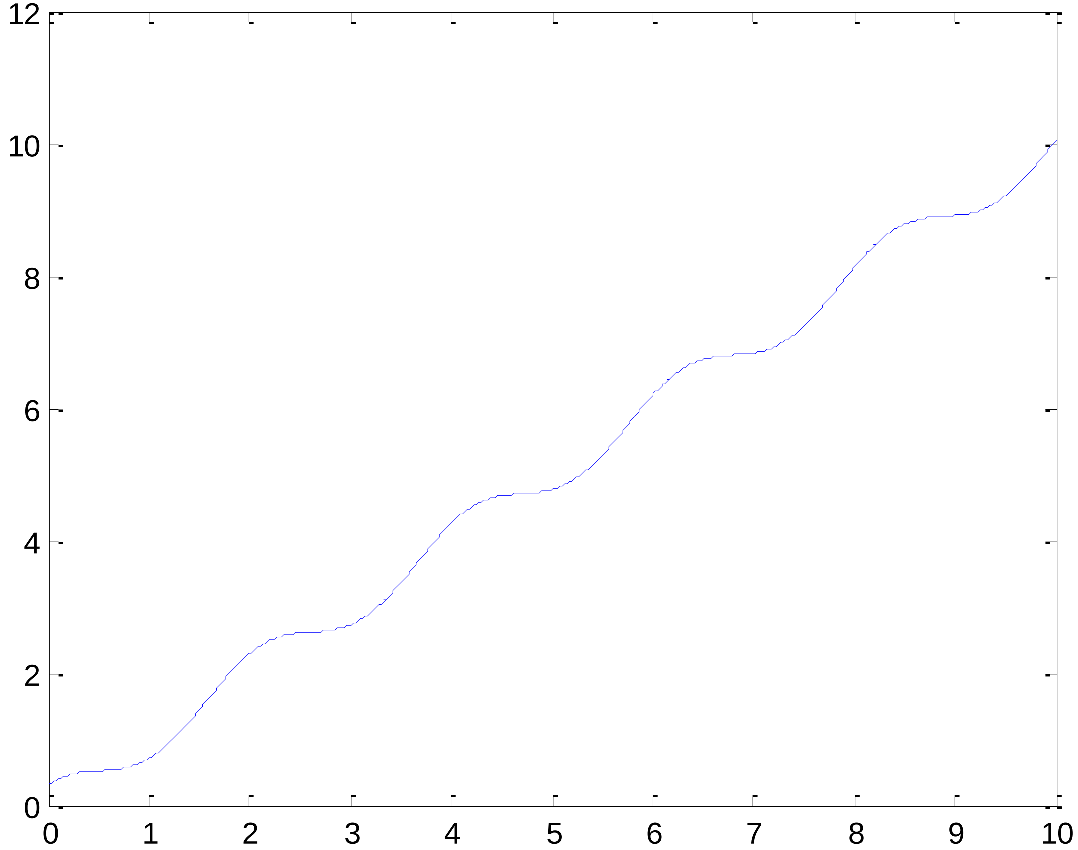
$\cos(20\pi t)+0.2*\cos(70\pi t)$



$\cos(20\pi t)+0.08*\cos(70\pi t)$

One or two frequency?

Is it a Trend?



$$x(t) = t + 0.3\cos(3t)$$

$$x'(t) = 1 - 0.9\sin(3t) > 0, \forall t$$

No local extrema

One or two frequencies?

- Gabriel Rilling and Patrick Flandrin [2008]

$$x(t) = \cos(2\pi t) + a\cos(2\pi f t + \phi),\ f \in (0,1)$$

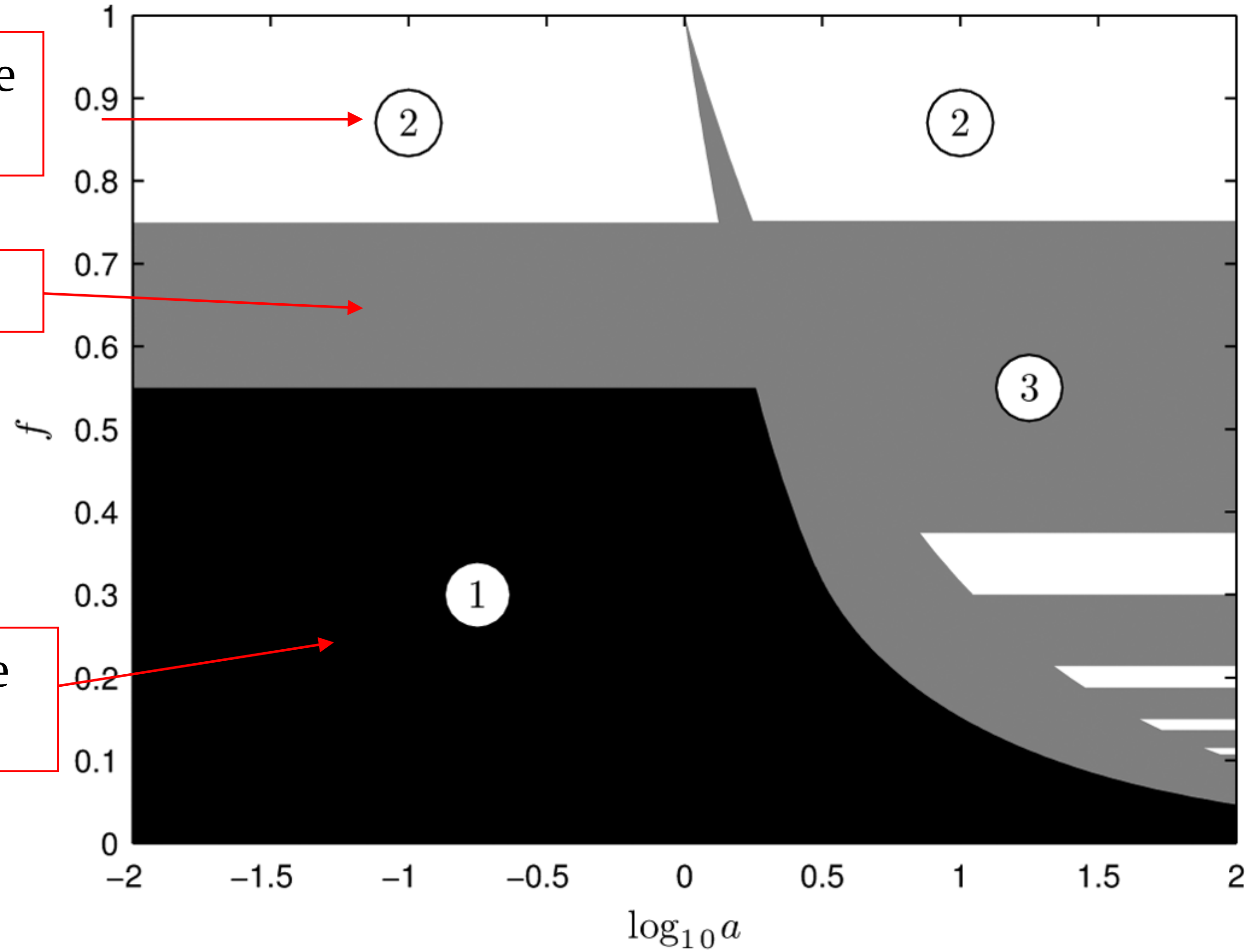
  - $af < 1$ the extrema rate is exactly the same as that of the HF component
  - $a^2 f > 1$ the extrema rate is exactly the same as that of the LF component

One or two frequencies?

considered as a single waveform

does something else

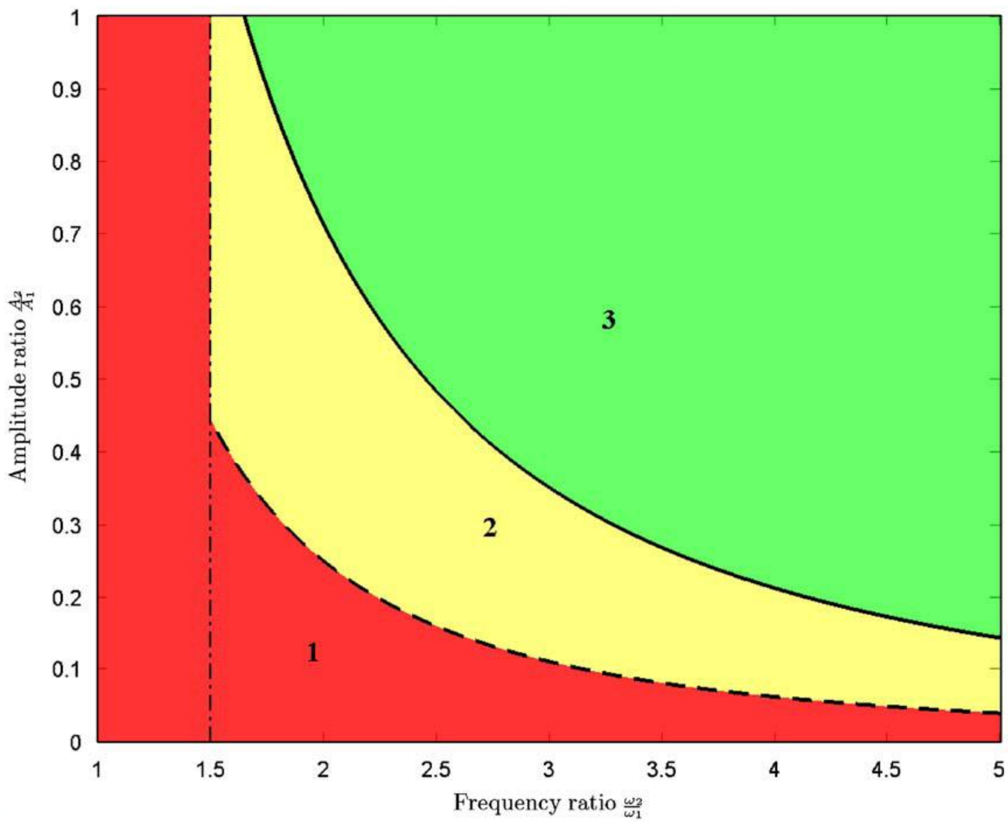
two components are well separated



One or two frequencies?

- $f(t) = A_1 \cos(2\pi f_1 t) + A_2 \cos(2\pi f_2 t)$

$f_2 > f_1$ 、 $A_1 > A_2$



- Range 1 : [12]

  They are as a single waveform.

  $$\left(\frac{A_2}{A_1}\right) \le \left(\frac{\omega_1}{\omega_2}\right)^2 \qquad \frac{\omega_2}{\omega_1} \le 1.5$$

  or

- Range 2 : [12]

  Decomposition requires several sifting iterations for close frequency harmonics.

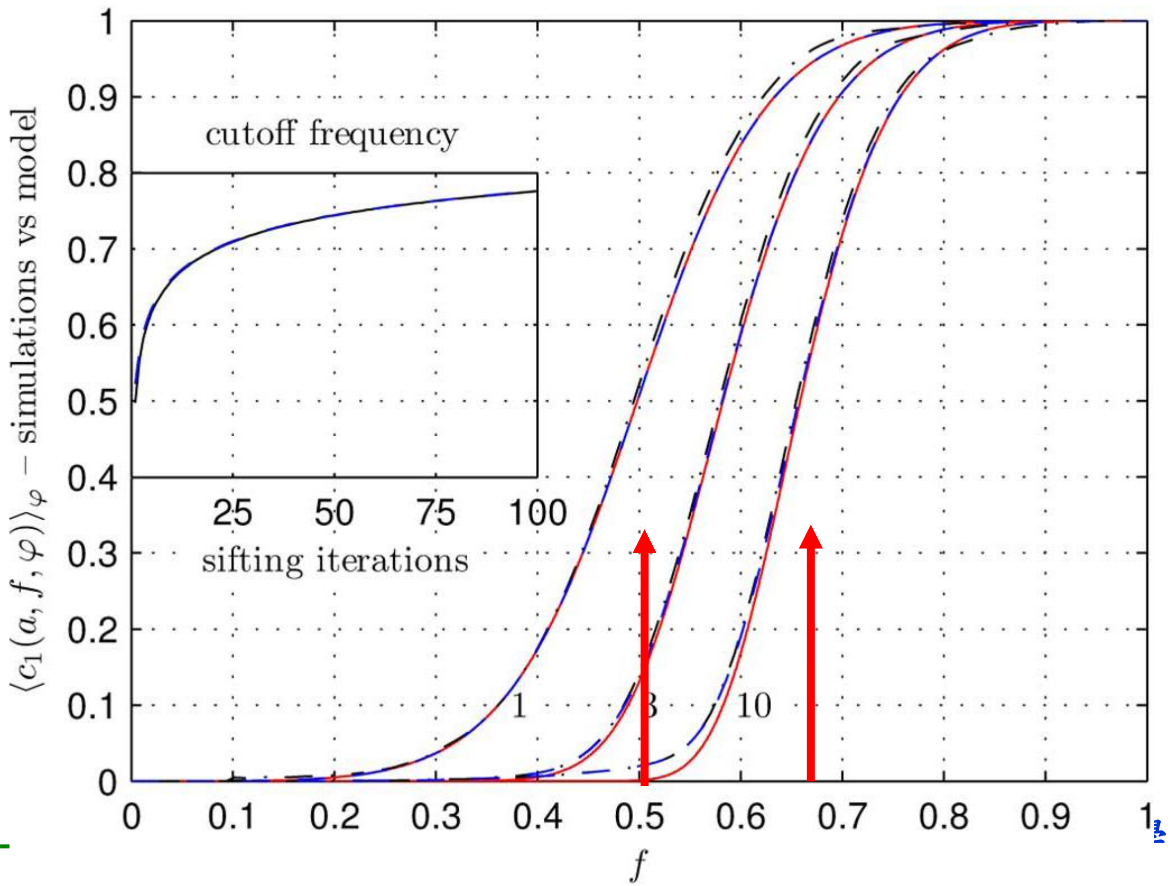
  $$\left(\frac{\omega_1}{\omega_2}\right)^2 \le \frac{A_2}{A_1} \le 2.4\left(\frac{\omega_1}{\omega_2}\right)^{1.75}$$

  $$\frac{A_2}{A_1} \ge 2.4\left(\frac{\omega_1}{\omega_2}\right)^{1.75}$$

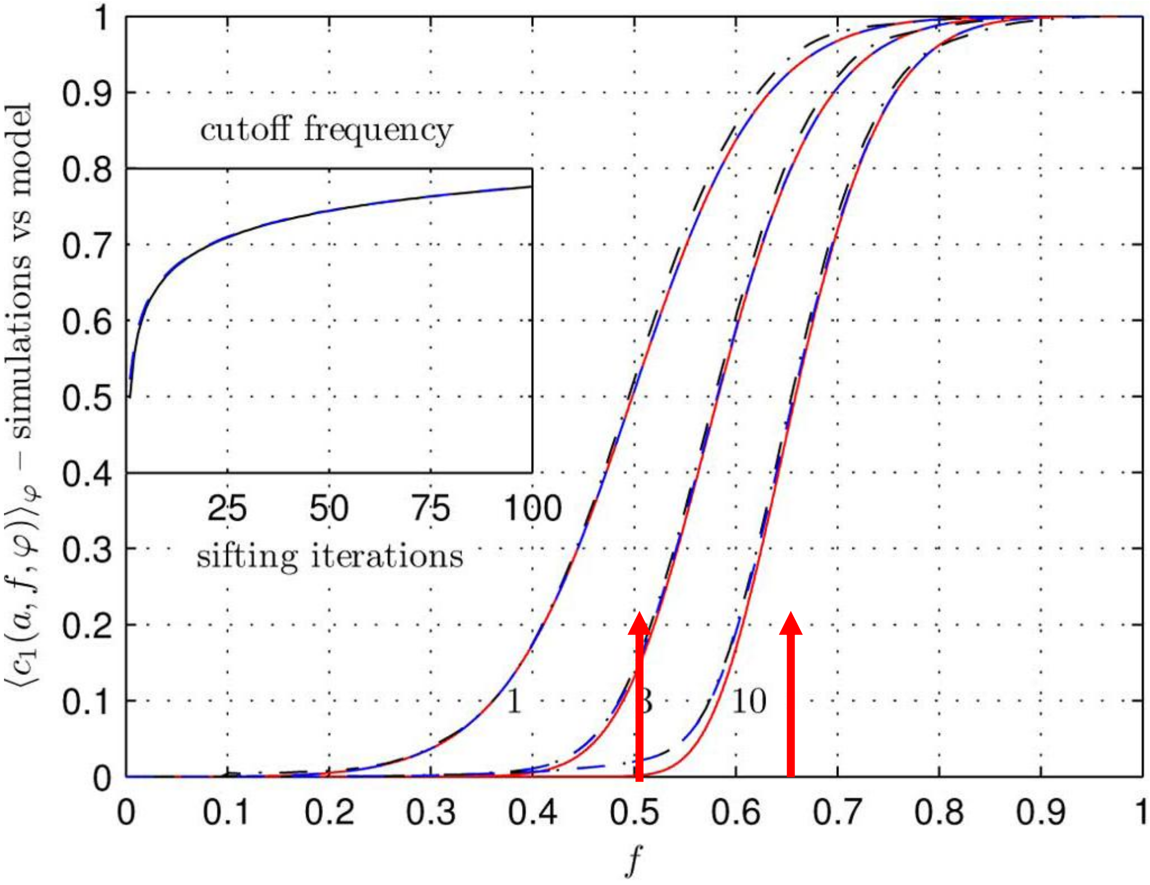
- Range 3 : [12]

The Effect of Iteration

- The larger the iteration number of EMD is, The narrower the transition zone is.



The Effect of Iteration Number

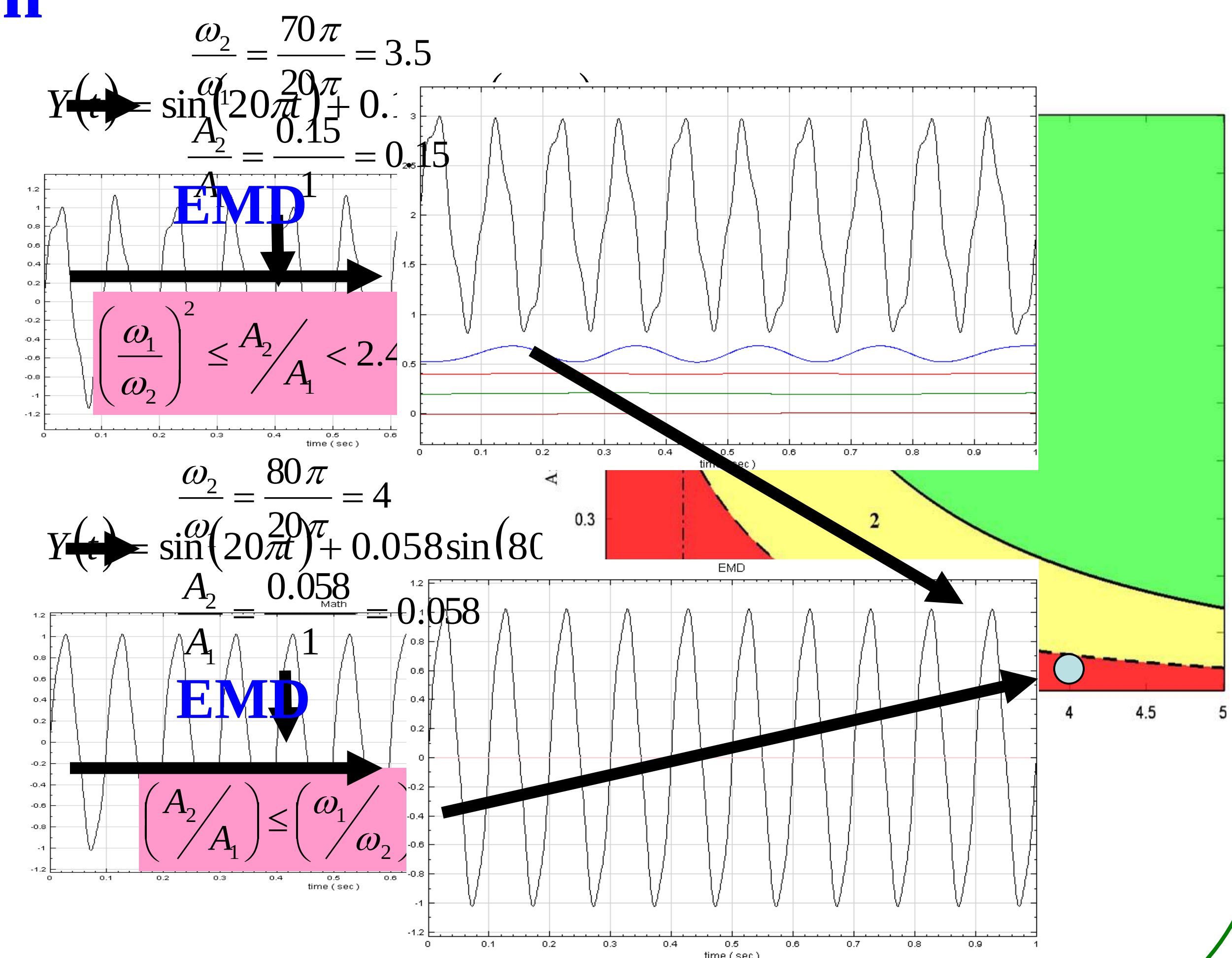


The Effect of Iteration Number

- E. Huang [2010]
  - Both the empirical and the theoretical results point to the asymptotic state of infinite number of sifting iterations. The empirical result further established the processes how the final IMFs of constant amplitude are reached.

Differentiation Operator Method

- **The simulation signal**



Differentiation Operator Method

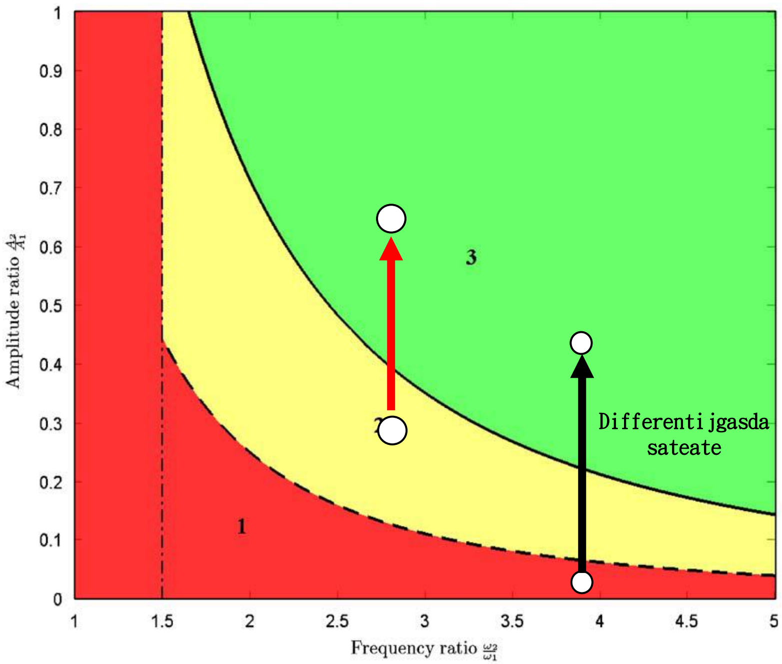
- Differential operator
  - Signal

$$f(t) = A_1 \cos(2\pi f_1 t) + A_2 \cos(2\pi f_2 t)$$

$$f_2 > f_1 \qquad A_1 > A_2$$

  - amplitude ratio

$$X_1(t) = \frac{A_2}{A_1}$$



  - Differential signal

$$f(t) = -f_1 A_1 \sin(2\pi f_1 t) - f_2 A_2 \sin(2\pi f_2 t)$$

  - amplitude ratio

$$X_2(t) = \frac{f_2 A_2}{f_1 A_1} = f\,\frac{A_2}{A_1}$$

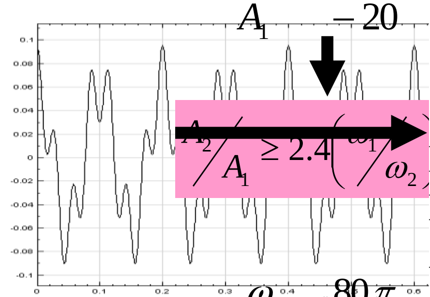
Differentiation Operator Method

- Differential signal

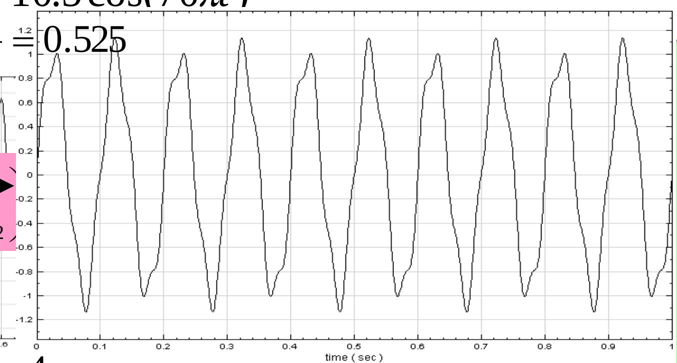
$$\frac{\omega_2}{\omega_1} = \frac{70\pi}{20\pi} = 3.5$$

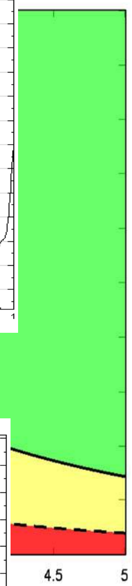
$$Y'(t) = 20\cos(20\pi t) - 10.5\cos(70\pi t)$$

$$\frac{A_2}{A_1} = \frac{-10.5}{-20} = 0.525$$



$$\frac{A_2}{A_1} \geq 2.4\left(\frac{\omega_1}{\omega_2}\right)$$

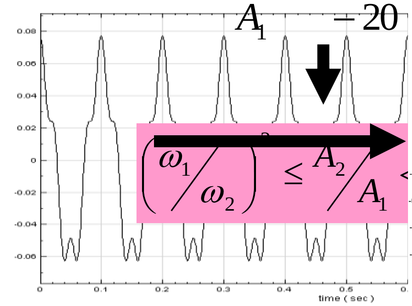




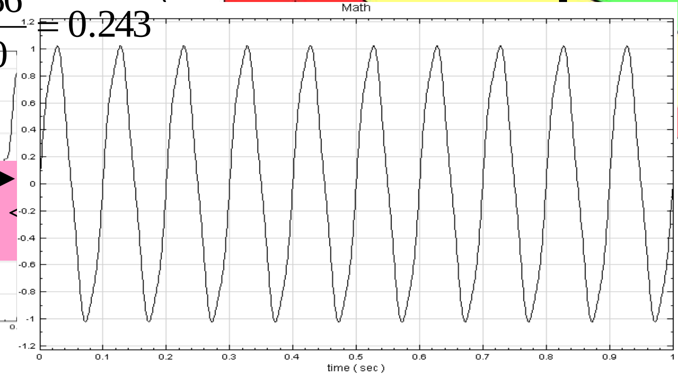
$$\frac{\omega_2}{\omega_1} = \frac{80\pi}{20\pi} = 4$$

$$Y'(t) = 20\cos(20\pi t) - 4.86\cos($$

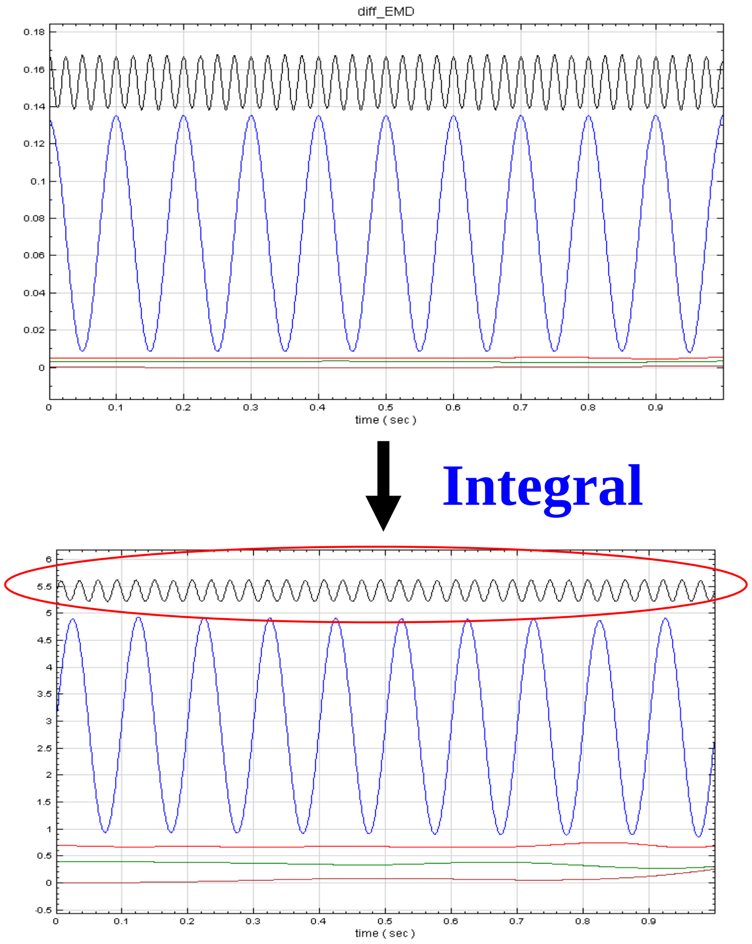
$$\frac{A_2}{A_1} = \frac{-4.86}{-20} = 0.243$$



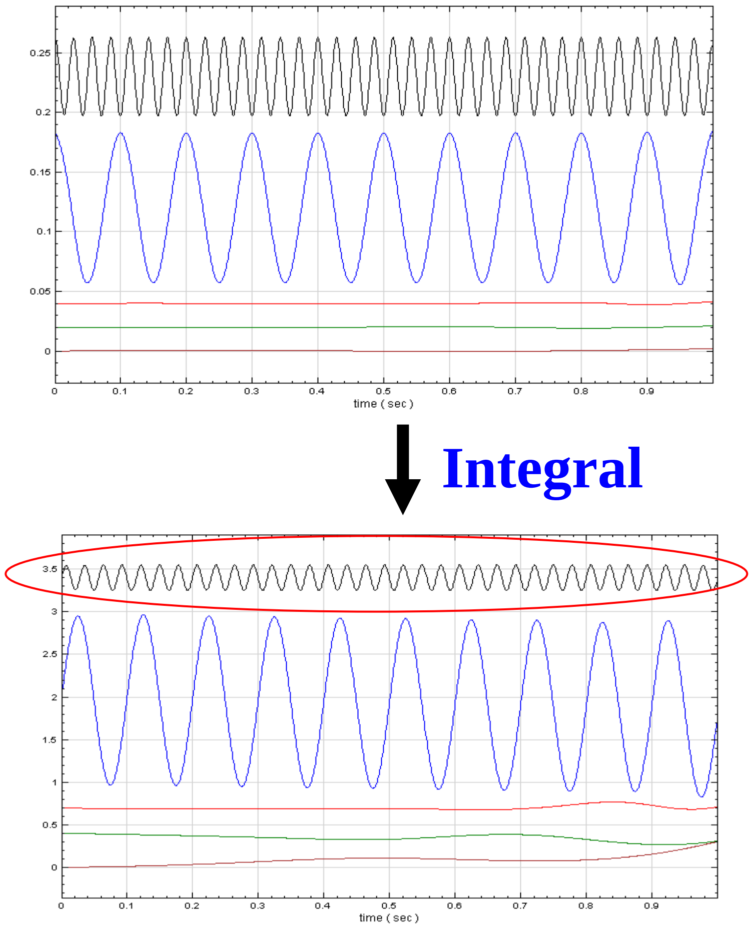
$$\left(\frac{\omega_1}{\omega_2}\right) \leq \frac{A_2}{A_1}$$



Differentiation Operator Method



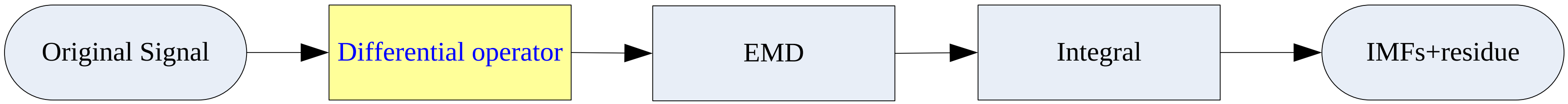
(a) Signal 1



(b) Signal 2

Differentiation Operator Method

- Flow Chart



- Disadvantage:
  - **Integration of IMF is not always a IMF**

OEMD

- Oblique EMD
  - Zhijing Yang , Lihua Yanga, Chunmei Qing [2010]

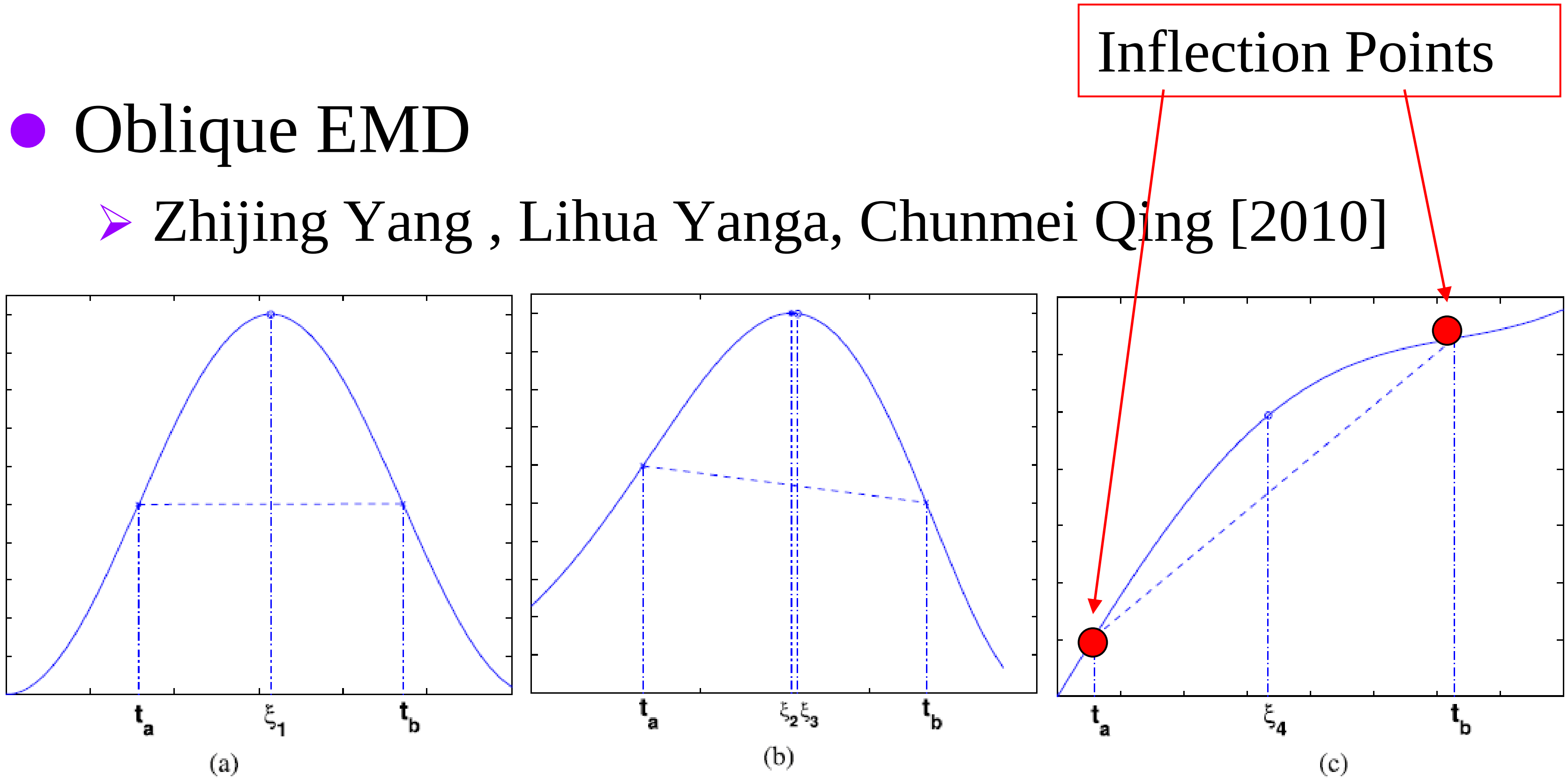


Fig. 4. (a) The oblique extremum point ξ_1 (circle) is an extremum point since $f(t_a) = f(t_b)$; (b) the oblique extremum point ξ_3 (circle) is different from the extremum point ξ_2 (asterisk) since $f(t_a) \neq f(t_b)$; (c) ξ_4 (circle) is an oblique extremum point but there is no extremum point on (t_a, t_b).

Practical Consideration of differentiation

- Forward difference scheme have phase lag

$$x'(n) = \frac{1}{T}(x(n) - x(n-1))$$

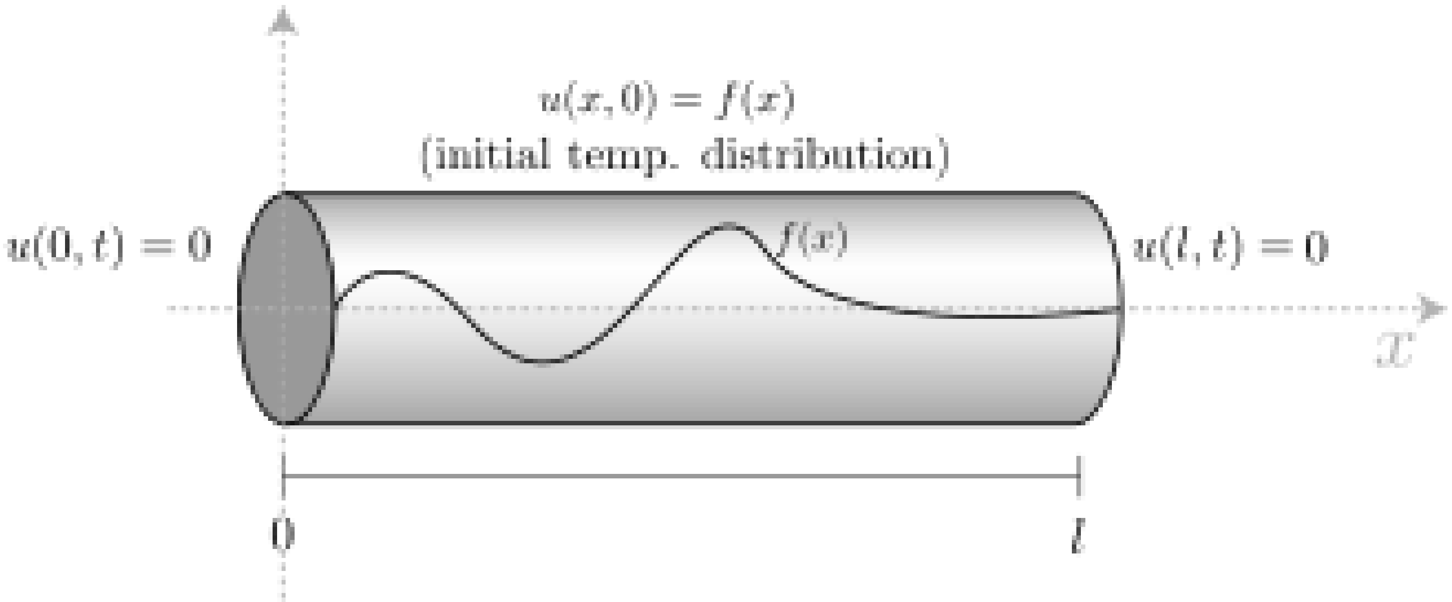
  - Location of the detected infection point is wrong.

- Central difference scheme (or high order central difference scheme) is suggested.

$$x'(n) = \frac{1}{T}(x(n+1) - 2x(n) + x(n-1))$$

  - Central difference is a zero phase filter → no phase lag.

Iterative Gaussian Filter



$$\begin{cases} u_t = k u_{xx}, -\infty < x < \infty, 0 < t < \infty \\ u(x,0) = f(x) \end{cases}$$

$$u(x,t) = \frac{1}{\sqrt{4\pi k t}} \int \exp\left(-\frac{(x-y)^2}{4kt}\right) f(y)dy$$

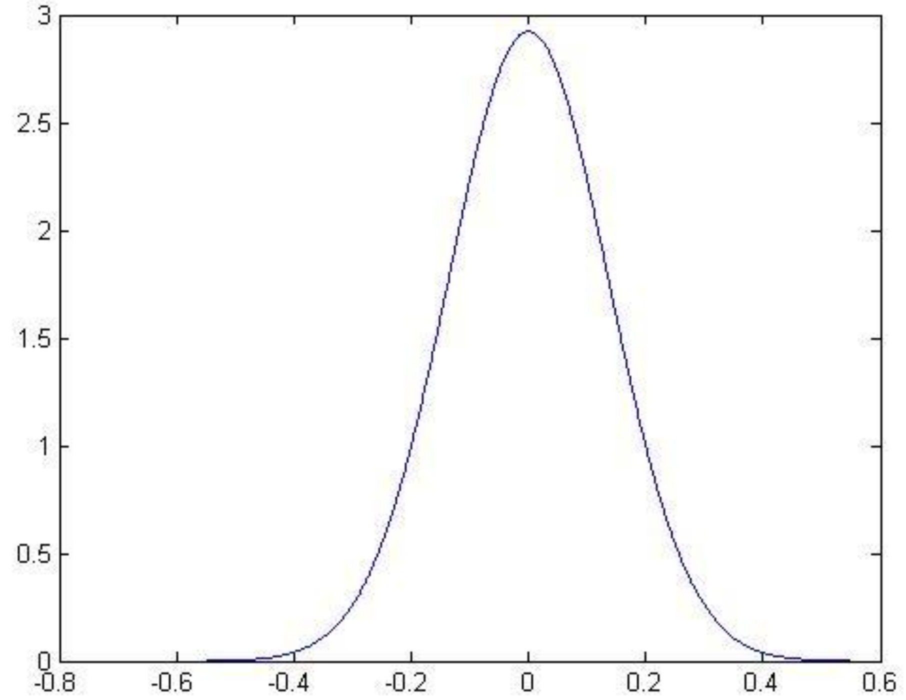
$$= \left(\frac{1}{\sqrt{4\pi k t}} \exp\left(-\frac{x^2}{4kt}\right)\right) * f(x)$$

Gaussian Window

- Time domain： $g(\sigma,t)=\frac{1}{\sqrt{2\pi\sigma}}e^{-\frac{t^2}{2\sigma^2}}$
- Frequency domain： $g(\sigma,t)\xrightarrow{fft}G(\sigma,\omega)=e^{-\frac{\omega^2\sigma^2}{2}}$

Frequency Response of Gaussian Filter

- Low pass filter
- Weighted averaging
- Poor Transition Zone

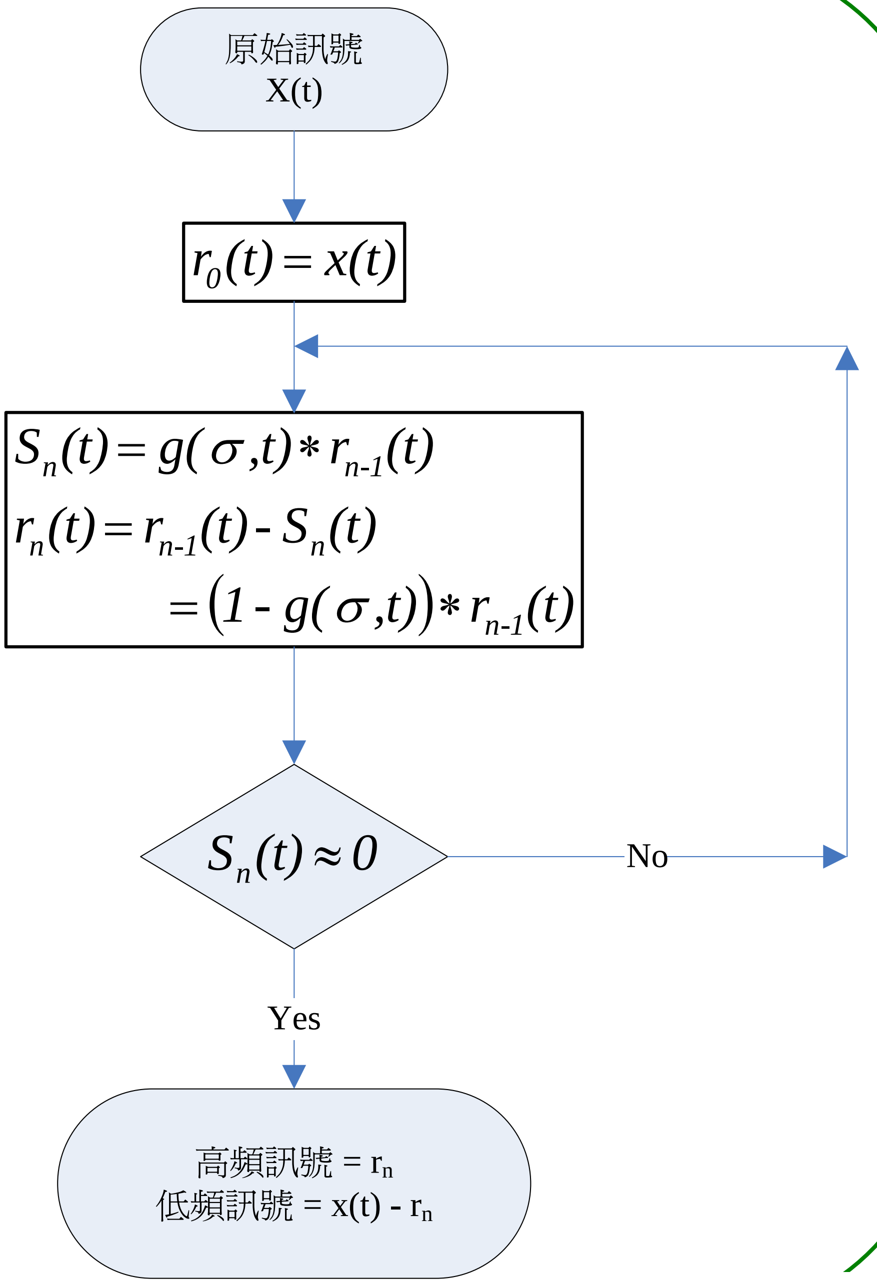


Survey of Iterative Gaussian Filter

- [1996] Iterative Smoothed Residuals: A Low-Pass Filter for Smoothing With Controlled Shrinkage
- [2008] Decomposition of one-dimensional waveform using iterative Gaussian diffusive filtering methods

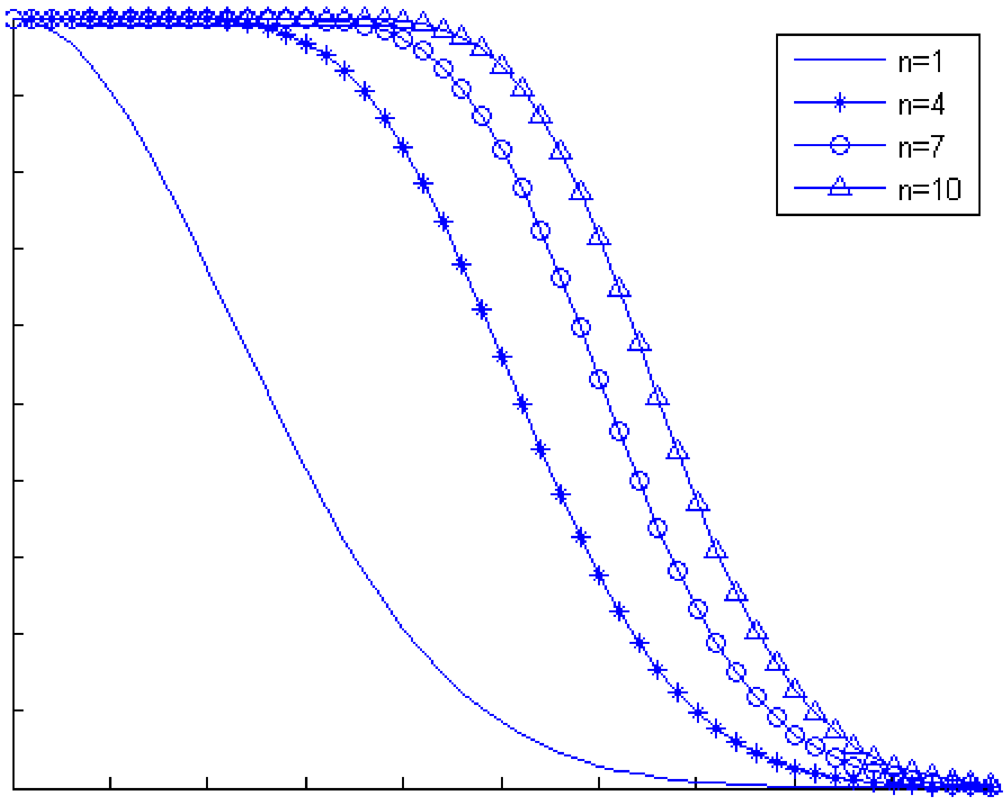
Iterative Gaussian filter

for signal decomposition



N次迭代高斯濾波器

- The larger the iteration number is, The narrower the transition zone is.



Low pass iterative gaussian filter

$$S_1 = g(\sigma,t) * x(t)$$

$$S_2 = (2g(\sigma,t) - g(\sigma,t) * g(\sigma,t)) * x(t)$$

$$S_3 = (3g(\sigma,t) - 3g(\sigma,t) * g(\sigma,t) + g(\sigma,t) * g(\sigma,t) * g(\sigma,t)) * x(t)$$

$$\vdots$$

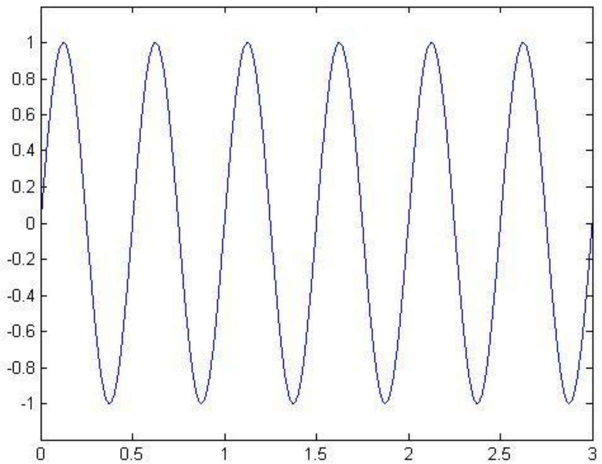
$$S_n = f(n,\sigma,t) * x(t)$$

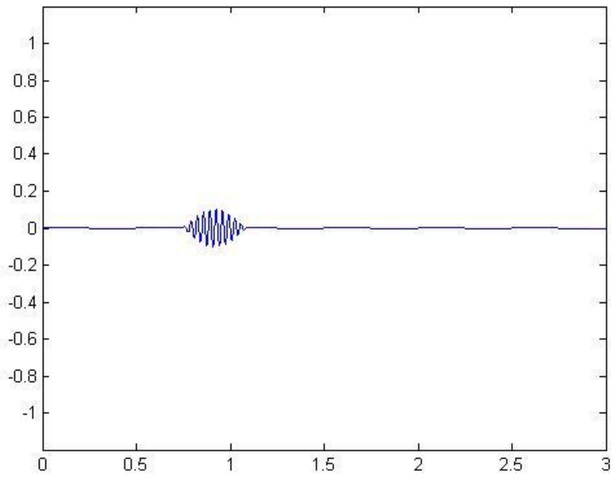
$$f(n,\sigma,t) = \sum_{i=1}^{n} (-1)^{i+1} \binom{n}{i} g(\sqrt{i}\sigma,t)$$

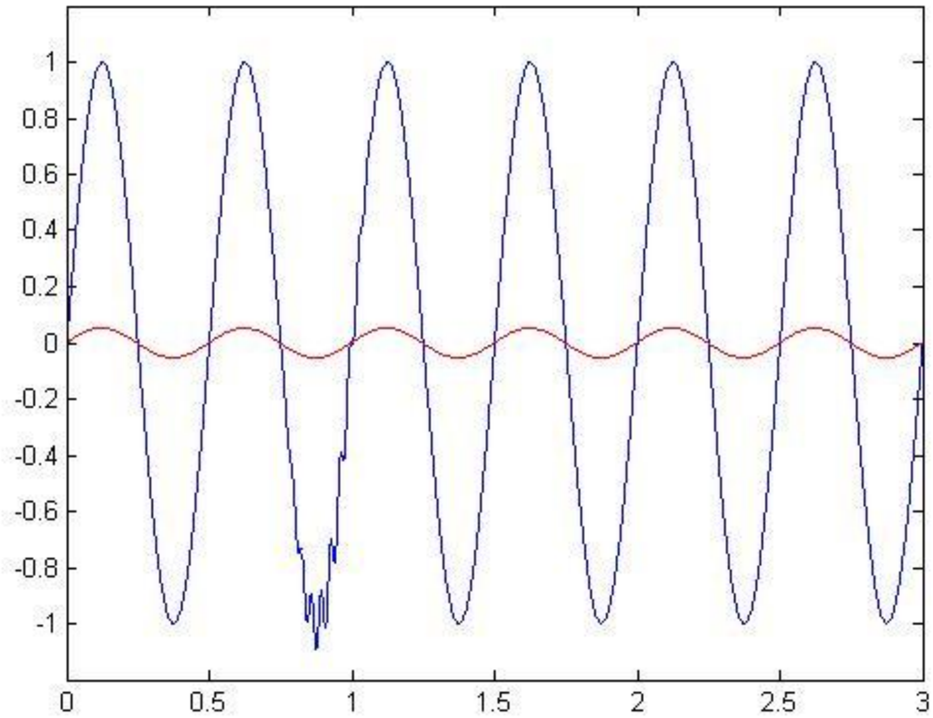
$$f(n,\sigma,t) \xrightarrow{fft} F(n,\sigma,\omega) = \sum_{i=1}^{n} (-1)^{i+1} \binom{n}{i} G(\sqrt{i}\sigma,t)$$

$$G(\sqrt{i}\sigma,t) = G(\sigma,t)^{i}$$

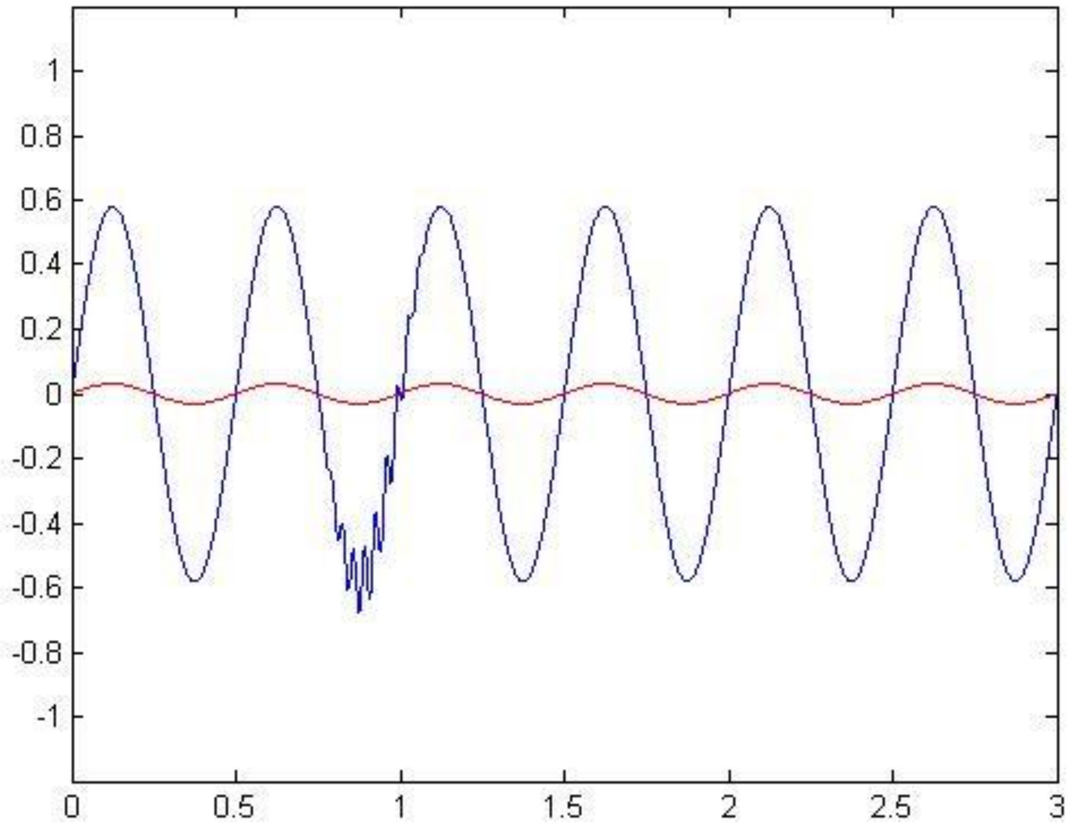
A Toy Example



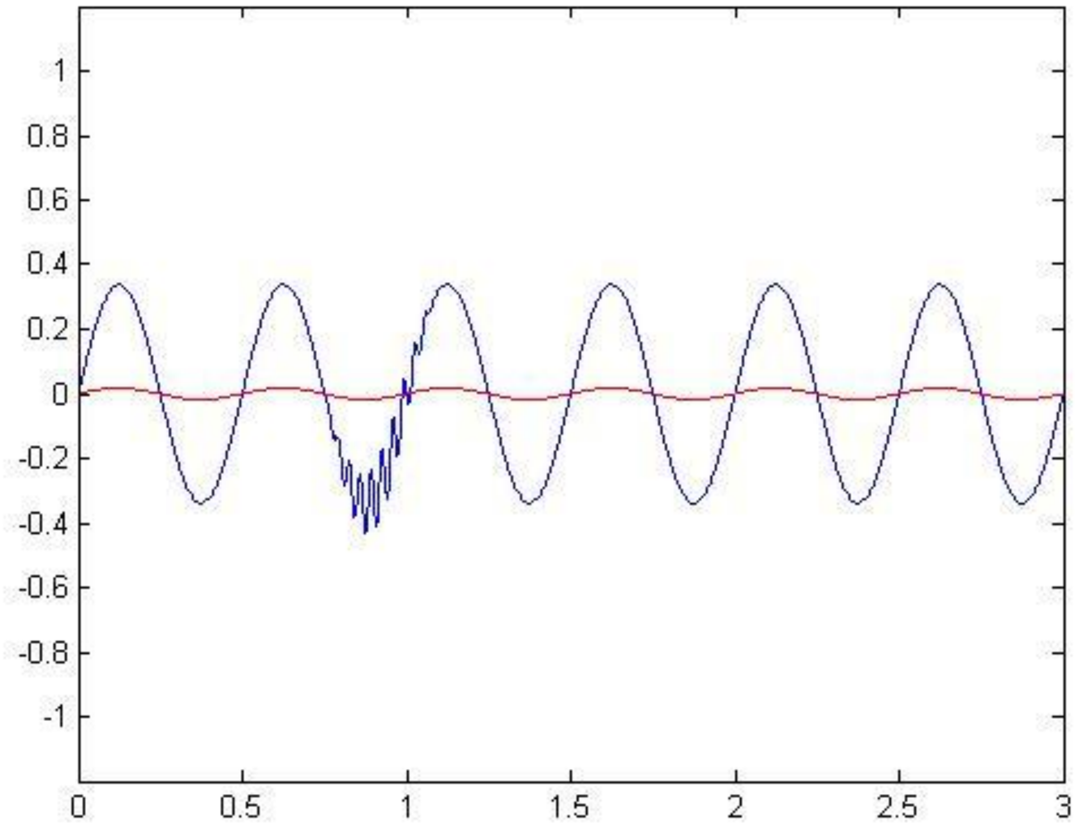




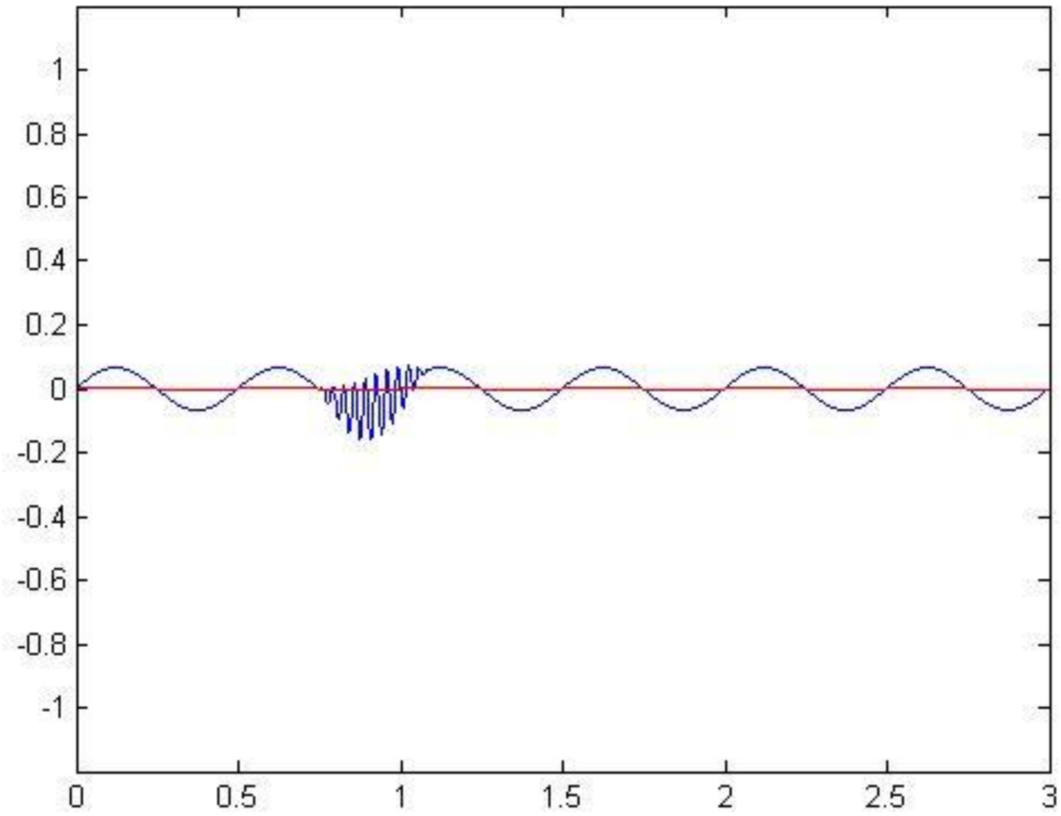
Number of Iterations: 10



Number of Iterations: 20

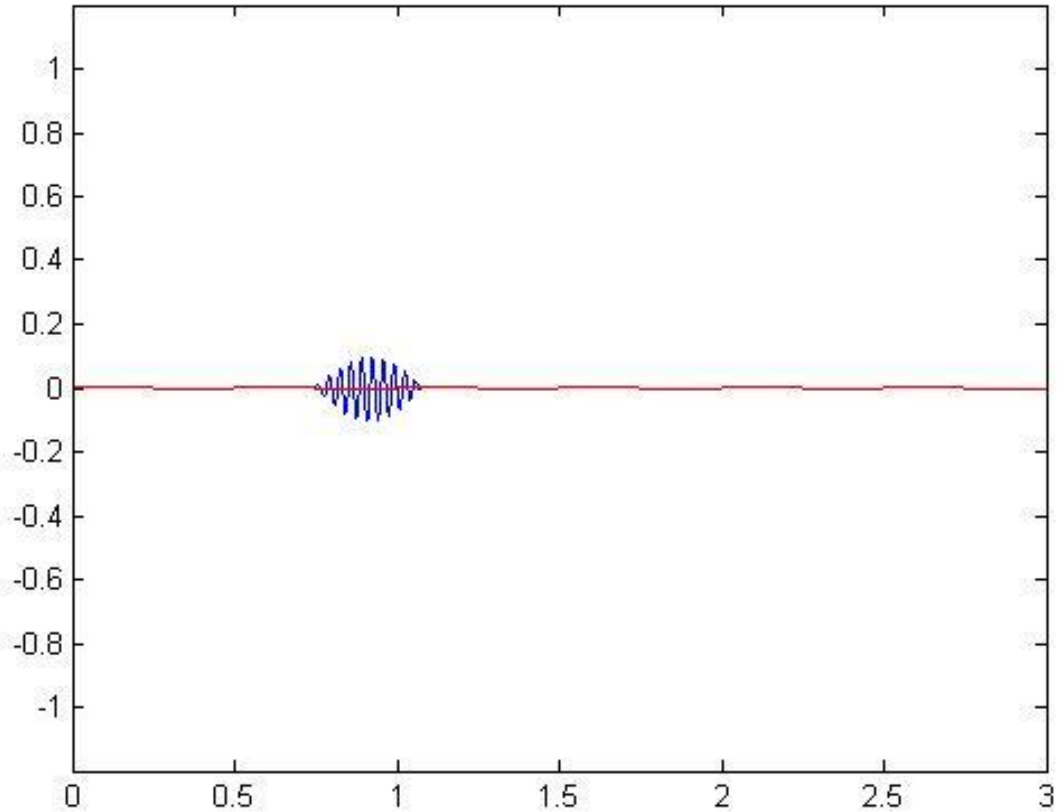


Number of Iterations: 50



Number of Iterations: 100

- Intermittency can be filtered out from the original signal



實務上的問題

- 遮罩長度會隨著其標準差增加，造成在時域上迴旋積的效率低落。
- Note:
  - Gaussian filter is a noncausal FIR filter
  - Noncausal → zero phase → extremas do not shift after applying the operation.
- Solution:
  - Using IIR filter to implement the gaussian window

IIR Gaussian filter

- IIR Gaussian filter
  - Ian T. Young, *Senior Member*, *IEEE*, Lucas J. van Vliet, and Michael van Ginkel [2002]

$$H_+(z) = \frac{1}{b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3}}$$

$$H_-(z) = \frac{B}{b_0 + b_1 z^{1} + b_2 z^{2} + b_3 z^{3}}.$$

$$b_0 = 1 \quad \text{scale} = (m_0 + q)\left(m_1^2 + m_2^2 + 2m_1 q + q^2\right)$$

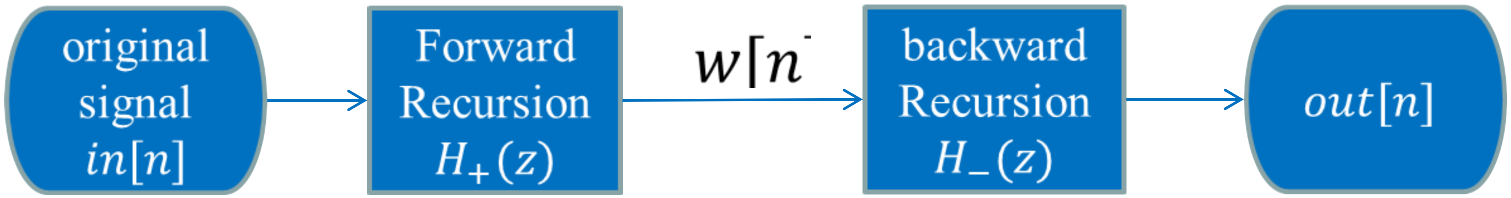
$$b_1 = -q\frac{\left(2m_0 m_1 + m_1^2 + m_2^2 + (2m_0 + 4m_1)q + 3q^2\right)}{\text{scale}}$$

$$b_2 = q^2\frac{(m_0 + 2m_1 + 3q)}{\text{scale}}$$

$$b_3 = -\frac{q^3}{\text{scale}}.$$

$$q(\sigma) = \begin{cases} -0.2568 + 0.5784\sigma + 0.0561\sigma^2, & \sigma < 3.556 \\ 2.5091 + 0.9804(\sigma - 3.556), & \sigma \geq 3.556. \end{cases}$$

Non Causal Gaussian filter

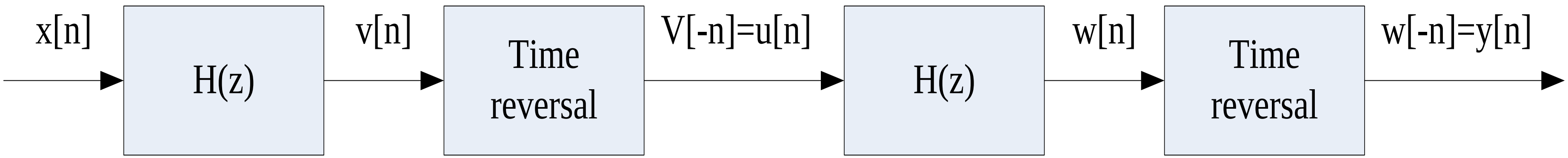


$$w[n] = \frac{(in[n] - (b_1 \bullet w[n-1] + b_2 \bullet w[n-2] + b_3 \bullet w[n-3]))}{b_0}$$

$$out[n] = \frac{(B \bullet w[n] - (b_1 \bullet out[n+1] + b_2 \bullet out[n+2] + b_3 \bullet out[n+3]))}{b_0}$$

Zero Phase Filter (noncausal filter)

Implementation of a zero-phase filtering scheme:



Fourier transforms:

$$V(e^{j\omega}) = H(e^{j\omega})X(e^{j\omega}) \qquad W(e^{j\omega}) = H(e^{j\omega})U(e^{j\omega})$$

$$U(e^{j\omega}) = V^{*}(e^{j\omega}) \qquad Y(e^{j\omega}) = W^{*}(e^{j\omega})$$

Combining the above equations, we obtain

$$Y(e^{j\omega}) = W^{*}(e^{j\omega}) = H^{*}(e^{j\omega})U^{*}(e^{j\omega}) = H^{*}(e^{j\omega})V(e^{j\omega})$$

$$= H^{*}(e^{j\omega})H(e^{j\omega})X(e^{j\omega}) = \left|H(e^{j\omega})\right|^{2} X(e^{j\omega})$$

[5] S. k. Mitra, “Digital signal processing”, *McGram.Hill international edition*.

Conclusions

- Suggestion
  - Signal Intermittency
    - ➔ Using Window Averaging Method to Remove
  - Improve the ability of signal decomposition for EMD
    - ➔ Using differential operator (OEMD)
- Open Question
  - EMD = iterative noncausal filter ?

Some Criticisms for EMD

- Orthogonality of IMFs
  - Necessary for Linear System
  - But… EMD is designed to deal with nonlinear system.
- Lack of mathematical foundation
- Instability of sifting process

What do you want?

- An universal tool

- A useful tool

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Since 2002
Department of Mechatronic Technology
National Taiwan Normal University

敬請批評與指教

Thanks for your attention!