

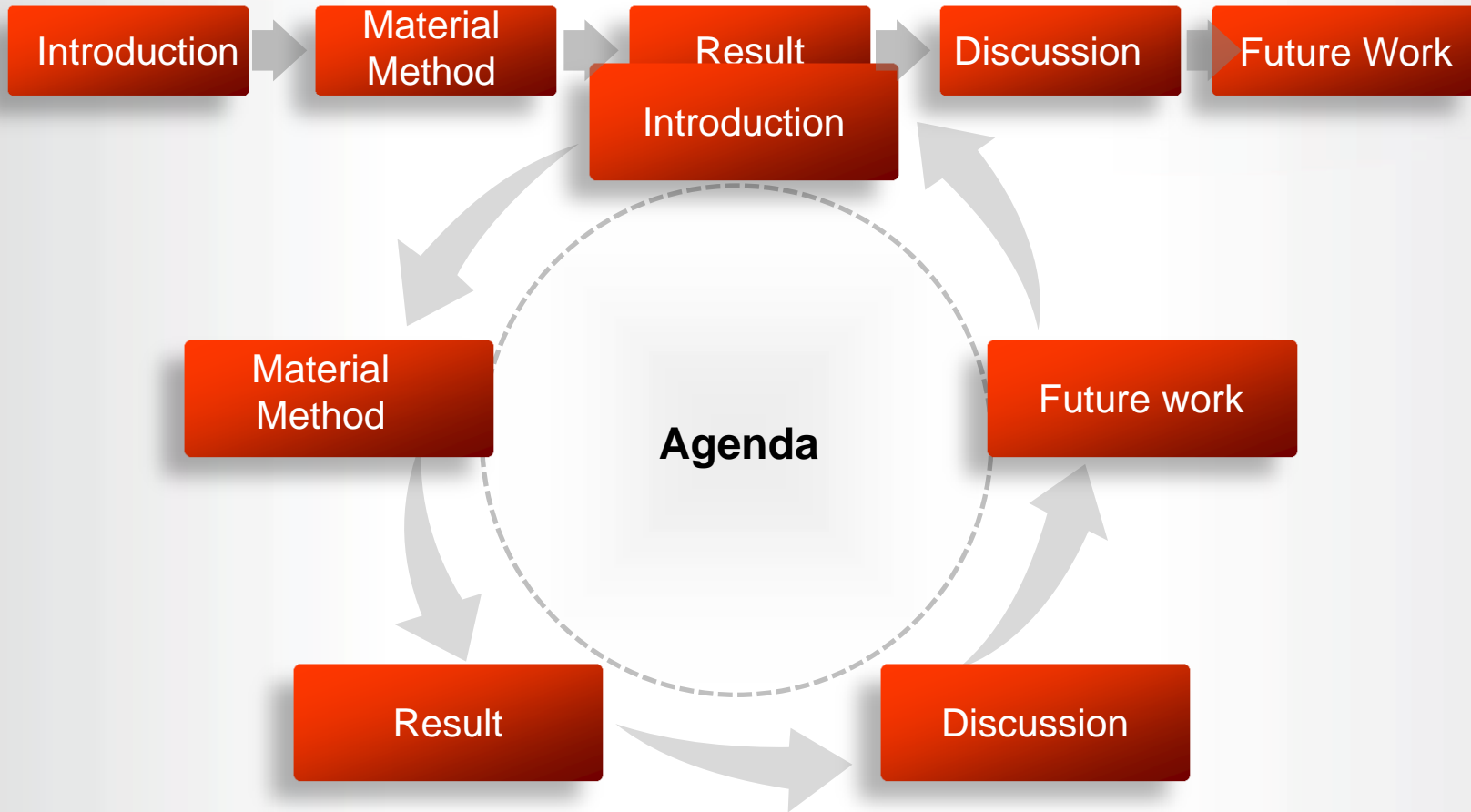


Noninvasive Method on ANS and 3D-Spectrogram Entropy Analysis

m

Speaker : Liao Wen Chien M.D.PhD

● Agenda



Top Secret

1

Background

2

Introduction

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Material

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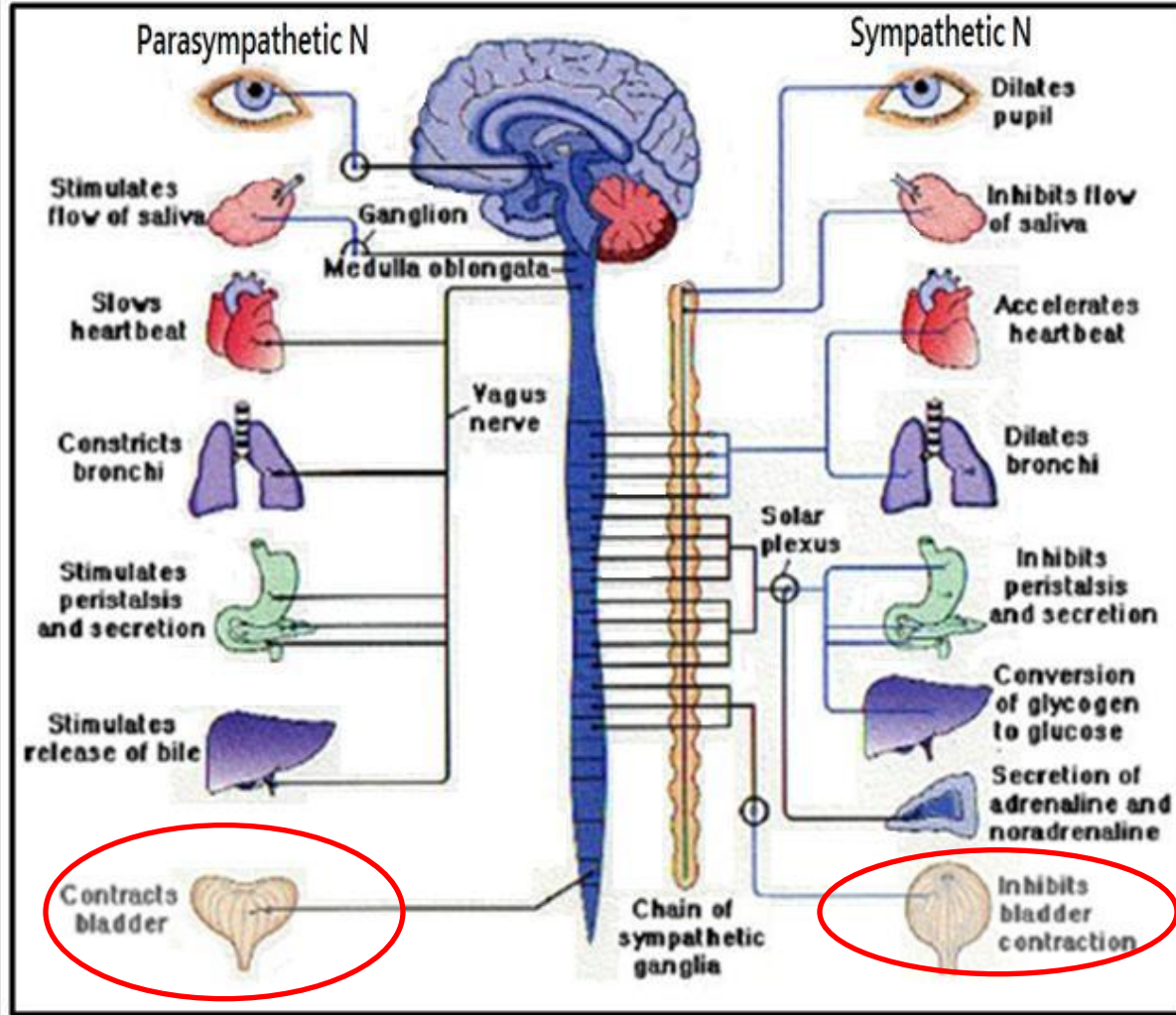
Result

6

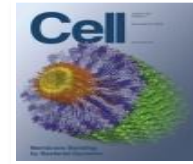
Discussion

Introduction

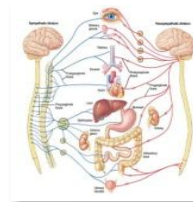
Autonomic Nervous System



Marco



Micro



Difficulty



Easy

项目背景
Object Background

Physiology of ANS

Otto Loewi F...

Syn

Heart

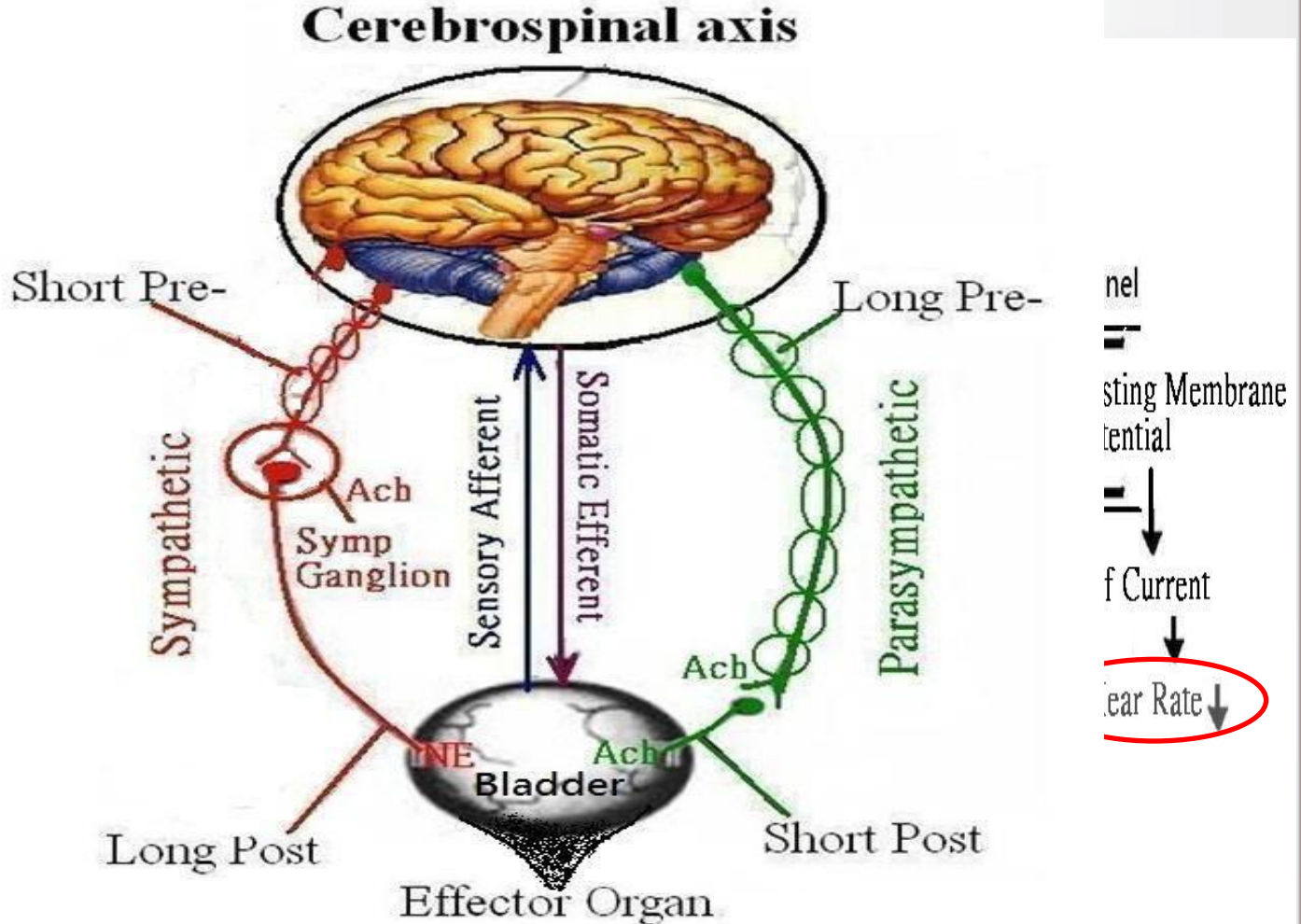
↑ Heart Rate

↑ Force Velocity

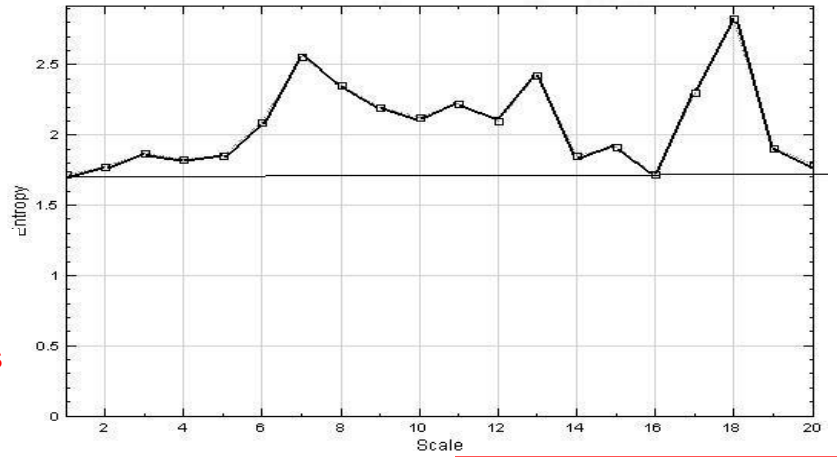
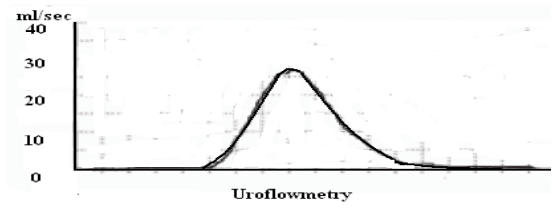
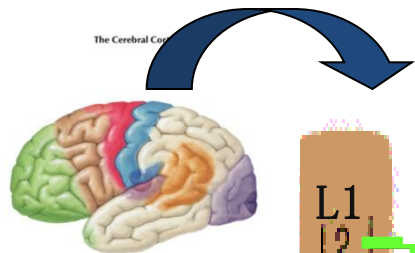
↑ Depolarization Inact

↑ If Current

↑ Cross-bridge c



nel
 ↓
 Resting Membrane Potential
 ↓
 f Current
 ↓
 Heart Rate ↓



Sacral Parasympathetic nucleus



Somatic (Pudendal n)

Parasympathetic n

Hypogastric n

Sympathetic ganglia

Inferior mesenteric ganglia

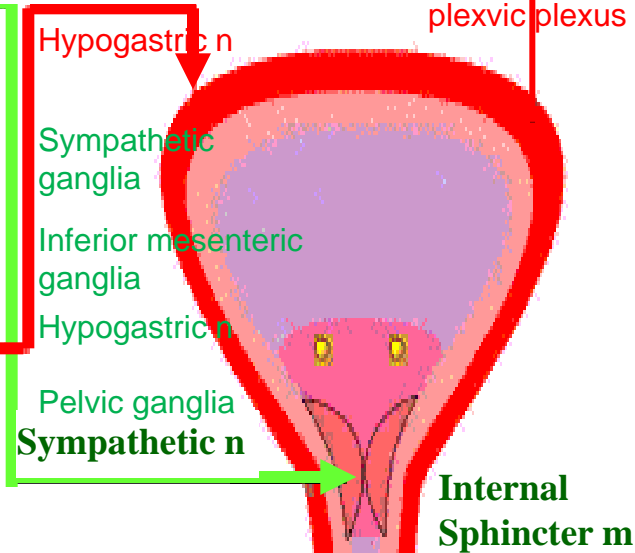
Hypogastric n

Pelvic ganglia

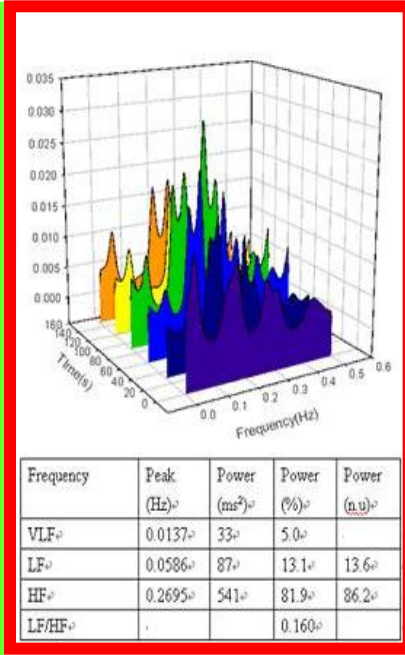
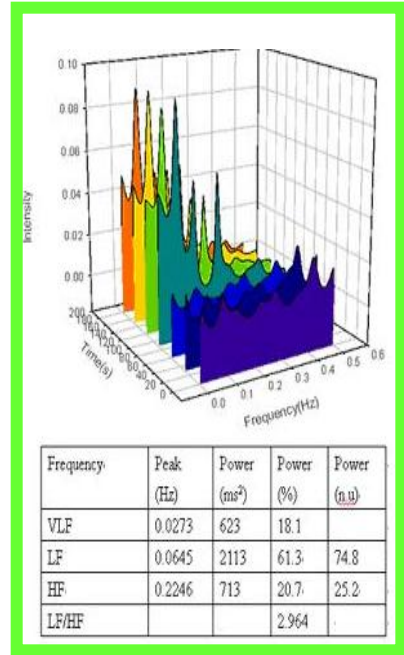
Sympathetic n

Detrusor m

plexicplexus



External Sphincter m



Introduction What is **Overactive Bladder** ?

2002 ICS Terminology

Urgency, with or without urge **incontinence**, usually with frequency and nocturia, in the absence of pathologic or metabolic factors that would explain these symptoms



Urinary
Frequency

Urgency

Urge
Incontinence

Nocturia

ICS=International Continence Society

Abrams P et al. Neuroyrol urodyn,2002,21,167-178

膀胱過動症自我檢查表

Self Screening for the Overactive Bladder

Frequent urination (> 8x per day) 小便頻密 (每天八次以上)

Night-time urination (> 2x per night) 夜尿 (每晚兩次以上)

Sudden need to urinate. 特發性尿意

Difficulty keeping it in. 忍尿困難

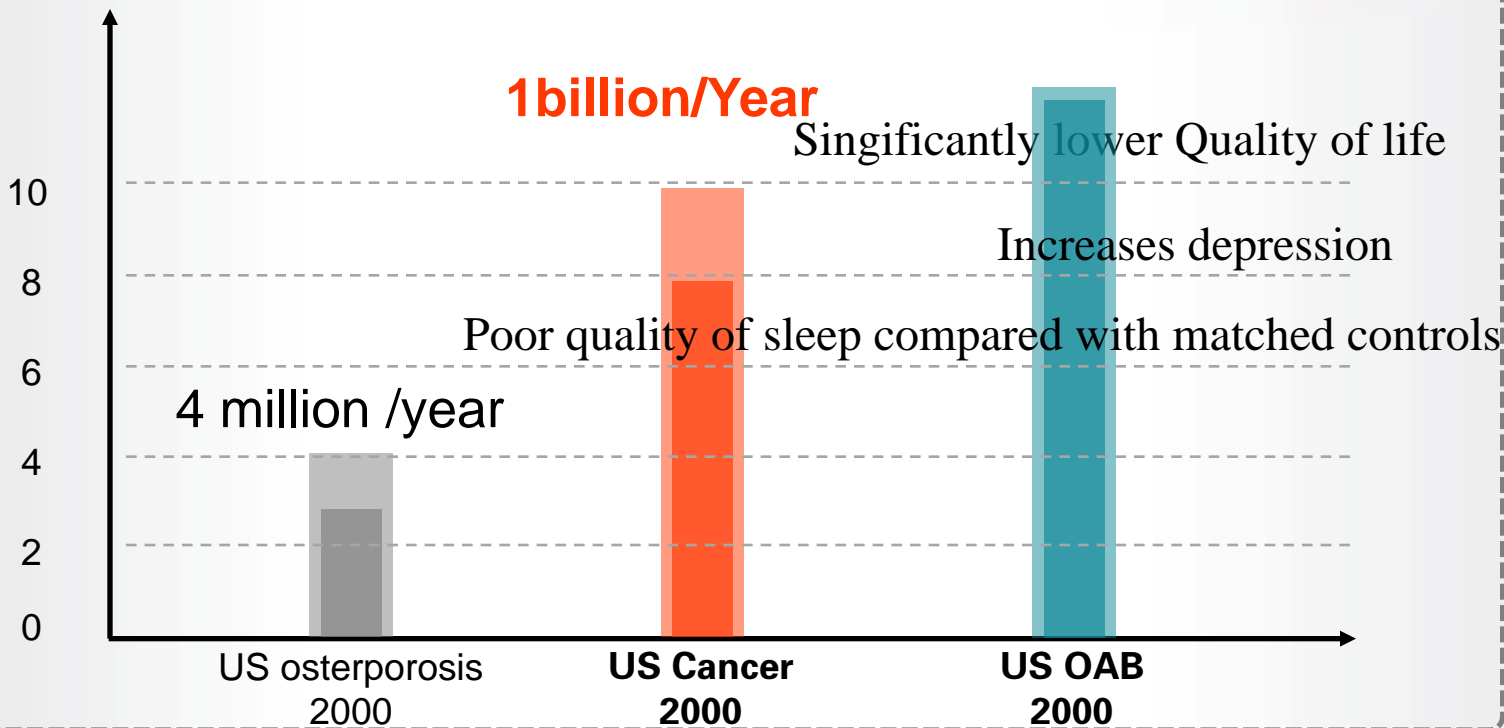
May have “accidents” when laughing, coughing or sneezing. 會因大笑、咳嗽、打噴嚏而漏尿

Incontinence: A lot A little 尿失禁: 大量 少量

● Cost Analysis

Overactive Bladder- Cost analysis

US 12 billion/year



Hu T-W,Wanger TH, Bentkover JD et al. Estimated economic costs of overactive bladder in the United States . Urology 2003; to the estimated costs of osteoporosis and gynecological 61:1123-1128

尿路動力學檢查儀

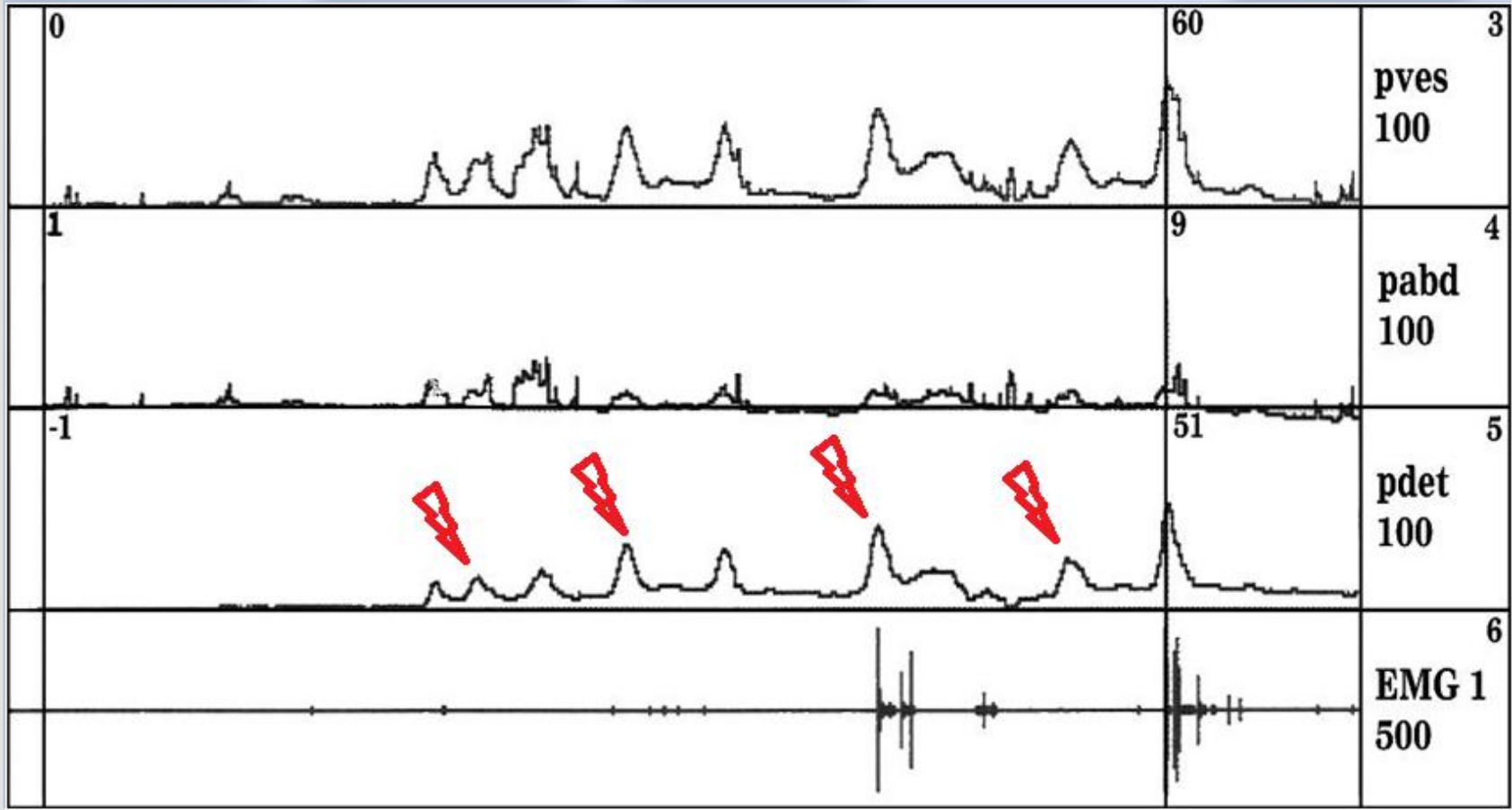
Introduction

Material
Method

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Future Work



Urinary Diary

1. In the 1st column mark an (x) every time you urinate into the toilet.
2. In the 2nd column, mark an (x) every time you accidentally leaked urine.
3. If an accident occurred, indicate the reason or circumstances surrounding the accident, for example, "coughed, bent over, sudden urge."
4. Under "Fluid Intake" describe the type (coffee, tea, juice, etc.) and amount (a cup, 1 quart, etc.).
5. Circle the time when you went to bed and when you got up in the morning.
6. Record number and type of pads used.
7. Under Notes write any additional information you would like to include. For example, type and dose of medication you may be on for your urinary incontinence.

TIME	URINATE IN TOILET	LEAKING ACCIDENT	REASON FOR ACCIDENT	FLUID INTAKE TYPE AMOUNT
6 a.m.				
7 a.m.				
8 a.m.				
9 a.m.				
10 a.m.				
11 a.m.				
12 Noon				
1 p.m.				
2 p.m.				
3 p.m.				
4 p.m.				
5 p.m.				
6 p.m.				

Evaluation of OAB and the Use of Questionnaire

1

OAB-q(2002) 4/25

2

**OAB Symptom Score(OABSS)
Y,Homma(2006) (7)**

3

**OAB Symptom Score(OABSS) JG
Blaivas(2006)**

4

**Patient Perception of Bladder
Condition(PPBC) (2006)**

5

OABSS in Taiwan Chinese (2008)

Material and Methods

Introduction



Material
Methods



Future Work

The OAB group consisted of **33 OAB adult women** 30–60 years, criteria (2003 International Incontinence Society), OABSS score >8.

All patients :urination detailed the time and volume

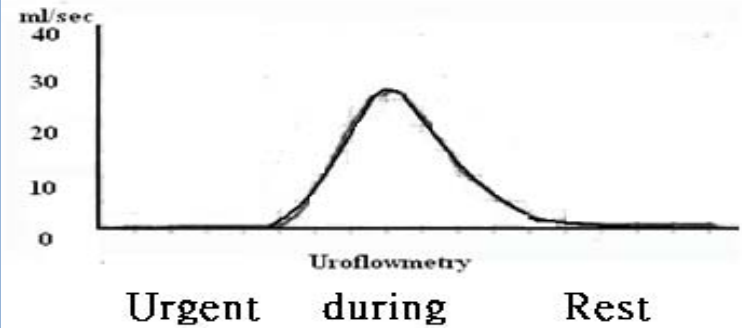
Medical history **questionnaire**

- (1)**medications** that can disrupt the ANS function, (e.g. urinary incontinence medications, beta-receptor-specific drugs, or antidepressants),
- (2)**pathological** conditions that may disrupt the ANS (e.g. coronary heart disease, neurological disease, diabetes and pelvic surgery, including hysterectomy).

OAB group :physical examination, ECG, urinalysis, uroflowmetry, and a pelvic ultrasound.

Material and Methods

Introduction

Material
Methods

A **non-invasive**, uninterrupted procedure voiding (**reflecting ANS control**) was used.

500 to 1000cc water was orally consumed by each subject (both control and test groups). When patients expressed a need to urinate, they were requested to hold their urine for at least 3min, and later, to urinate in a standard urocytometry chair.

Continuous ECG recording was made while the patient was holding her urine, to 5 minutes immediately **before** voiding, **during** voiding, and then for 5 minutes **post**-voiding. 1000Hz ECG (P-QRS-T) wave pattern measurements were taken during urination.

The amount of urine voided and flow rate was recorded

After voiding, a **bladder scan** was performed on each patient to determine the residual urine volume.

Material and Methods

Introduction



Material
Methods



Future Work

ECG data was converted into HRV data (Matlab)

Data was expressed as a mean \pm standard deviation(SD).

The **Mann-Whitney U test** , **Wilcoxon test** , and **Z tests** were used to compare electrophysiological data values, between OAB and control subjects. (**SPSS Inc, Chicago, Ill**) software

The relationship between various parameters was assessed using the **Pearson correlation** or **Spearman rank correlation** .

P values lower than 0.05 were considered significant.

Material and Methods

Introduction



Material
Methods



Future Work

(MatlaThe **Matlab** software program was used to calculate

Power spectrum density(PSD)

Multiscale entropy graphs

3D-spectrograms.

$$Z(t) = \sum_{j=1}^n a_j(t) e^{i2\pi \int f_j(t) dt}$$

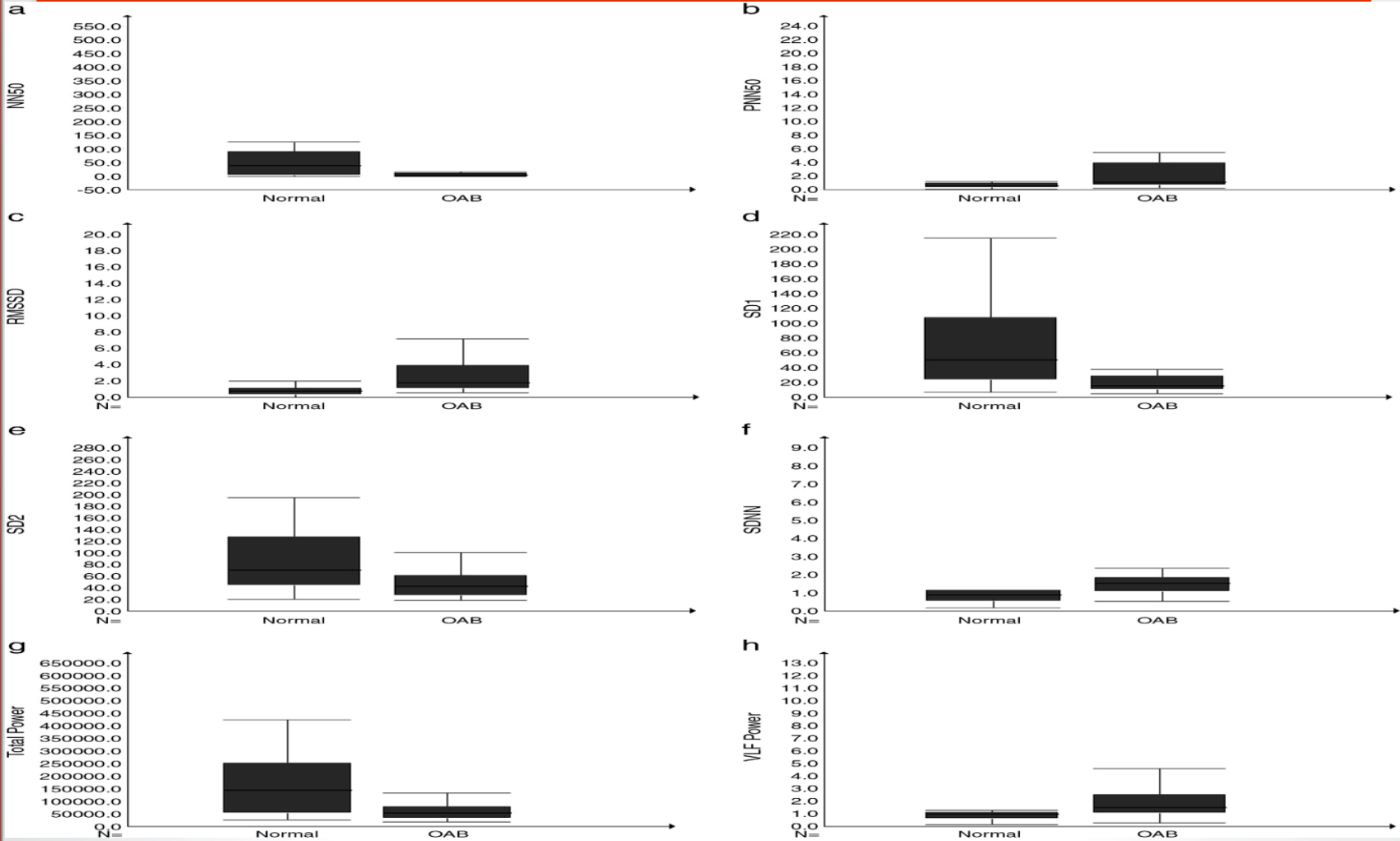
Each of these resulting data sets represents a different mathematical approach to **quantify** and display HRV data, and therefore **ANS activity**.

Each provides a specific fingerprint that distinguishes HRV between normal and OAB patients.

The **neuron signal normalization entropy graphs** were explained using the **Hilbert Hwang Transform (HHT)** equations:

$$h(\omega) = \frac{1}{T} \int_0^T H(\omega, t) dt$$

Statistics Analysis



Micturition 3D-Spectrogram Normal VS OAB

Introduction



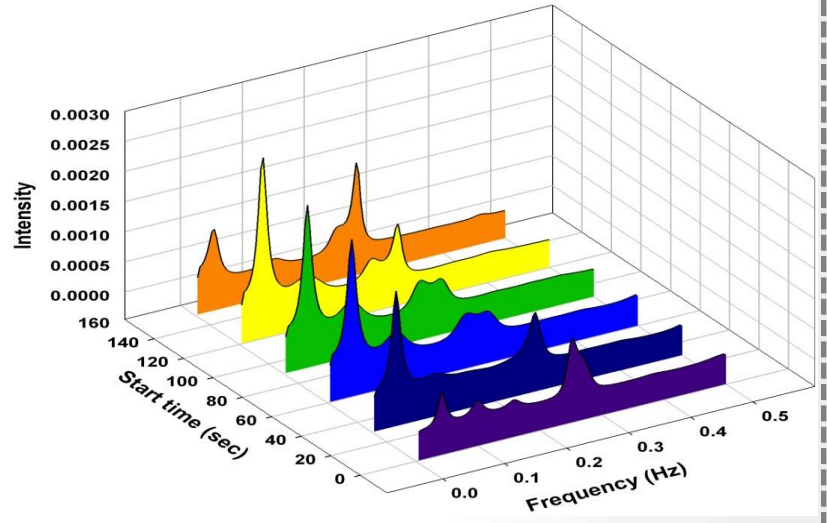
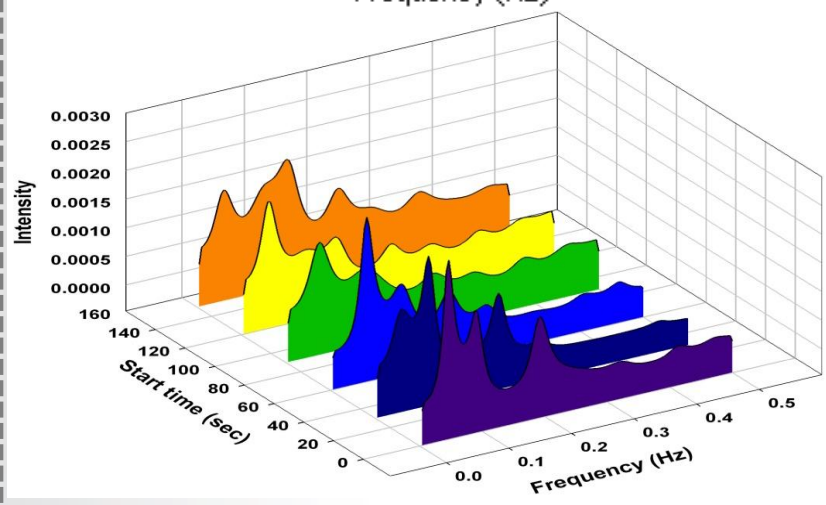
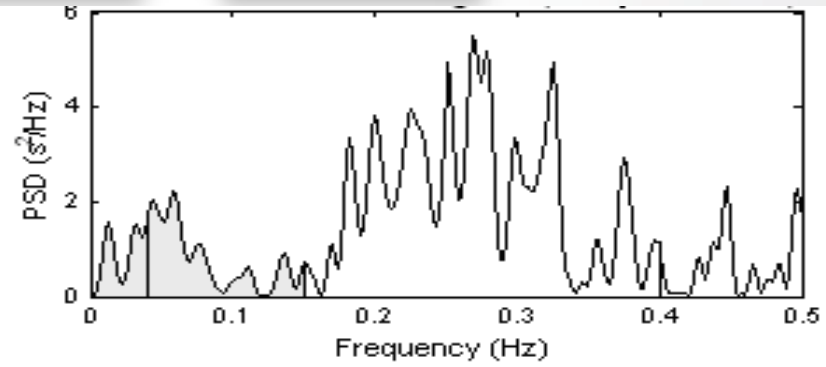
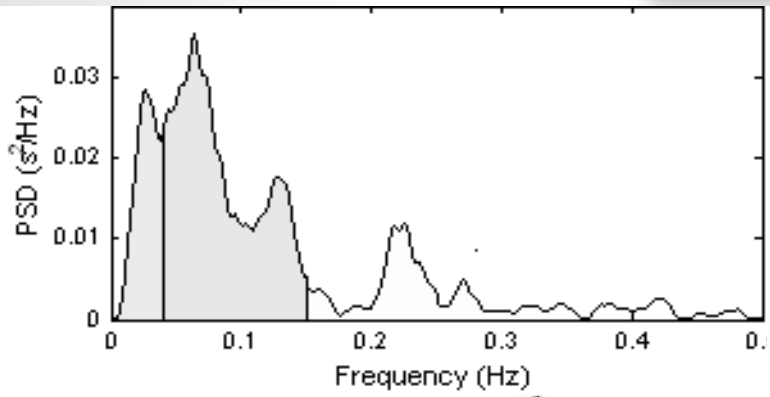
Method



Result



Discussion





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CLINICAL ARTICLE

A noninvasive evaluation of autonomic nervous system dysfunction in women with an overactive bladder

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ABSTRACT

Objective: To verify the hypothesis that a connection exists between overactive bladder (OAB) syndrome and a bladder-specific dysfunction of the autonomic nervous system (ANS). **Method:** An electrocardiogram recorded heartbeat cycles from the onset of urinary urgency to 5 minutes after voiding in 33 women with an overactive bladder and 176 controls. Power spectral density (PSD) analysis allowed to quantify heart rate variability (HRV), which is in relation to ANS function. Three-dimensional spectrograms and multiscale entropy graphs were used to display HRV values. **Results:** The differences between patients and controls were all significant in the time and frequency domains of HRV ($P < 0.05$), which suggests disturbances in bladder-specific ANS activity in women with OAB. **Conclusion:** By quantifying HRV data, PSD analysis provides a simple, noninvasive method of assessing disturbances in ANS activity and monitoring treatment in women with OAB. It can also be used to evaluate other neuronal conditions.

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1. Introduction

The overactive bladder (OAB) syndrome was defined by the International Continence Society as “urinary urgency, with or without urinary incontinence, usually with frequency and nocturia,” in the absence of local infection or other pathologic changes [1]. This syndrome, which is associated with a higher risk of falls, a lower quality of life and in treatment costs similar to those faced by women with osteoporosis or gynecologic cancers [4]. There are several theories for the cause of OAB, which may not be mutually exclusive [3,5–8]. Some are based on the anatomical or functional changes of the bladder activity, when the bladder contracts inappropriately, or in the parasympathetic fibers of the muscular layer of the body of the bladder (or bladder detrusor) [9,7]. Others are based on the sympathetic or parasympathetic fibers that stimulate the bladder to contract after the bladder has emptied, which cause the bladder to contract again before it is full; and these fibers are part of the autonomic nervous system (ANS) [10].

Quantifying the ANS activity is a difficult task. The most common method is the OABSS score (OABSS), which goes from 0 to 15, is subjective and therefore has limitations, and urodynamic investigations can be invasive and are time consuming. Other methods are more objective, and

noninvasive method of measuring the activity of the nerve fibers that control the urge to urinate (or micturition) and urination.

A direct relationship between heart rate variability (HRV) and parasympathetic effects has been reported [11,12], including in studies focusing on women with OAB or urinary stress incontinence [13] or children with nonsymptomatic nocturnal enuresis [14]. HRV and its components [15] have demonstrated that parasympathetic activity in the ANS can be evaluated by performing spectral density (PSD) analyses of HRV. The aim of the present study was to: (1) quantify HRV data by means of PSD analyses for all participants; (2) display the obtained values by means of 3-D spectrograms and multiscale entropy graphs; (3) examine the differences between the two groups of patients, as differences would indicate disturbances in ANS activity in women with OAB.

Materials and methods

The study was carried out from January 2004 through December 2006 with 33 women between 30 and 60 years of age seen for OAB at the outpatient clinic of the Department of Obstetrics and Gynecology of Taiwan Adventist Hospital. The 33 women were matched with 176 women without OAB. All women included in the study met the criteria for OAB as defined by the 2003 International Incontinence Society criteria [1] or had an OABSS score greater than 8. All had previously provided a urination pattern detailing the time and volume of each urination over at least 3 days. Further screening was the same for study patients and controls. It included answering a medical history questionnaire designed to exclude women treated with medications for urinary incontinence, drugs targeting beta receptors, antidepressants, or any

Reviewer : GENERAL COMMENTS
This is an excellent and highly original paper, with a valid and proven test.
The problem is that very few clinicians can understand it as is.

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 E-mail address: dr.liaowc@msa.hinet.net (W.-C. Liao).

Micturition 3D-Spectrogram Normal VS OAB

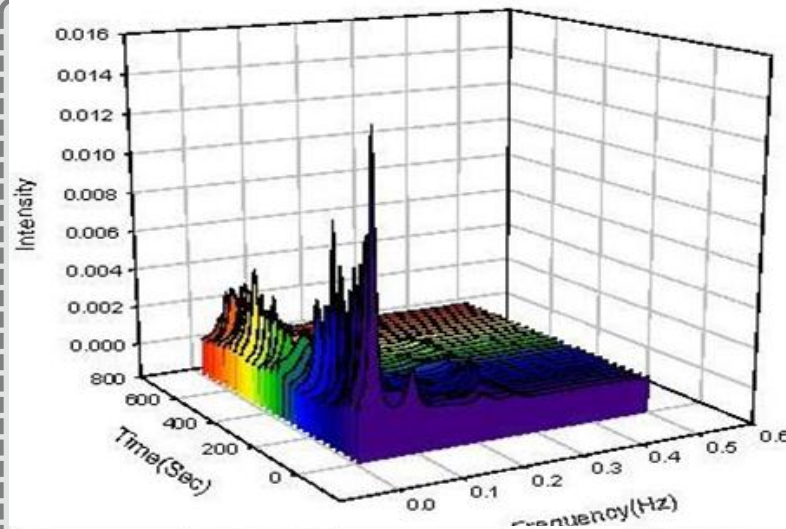
Introduction



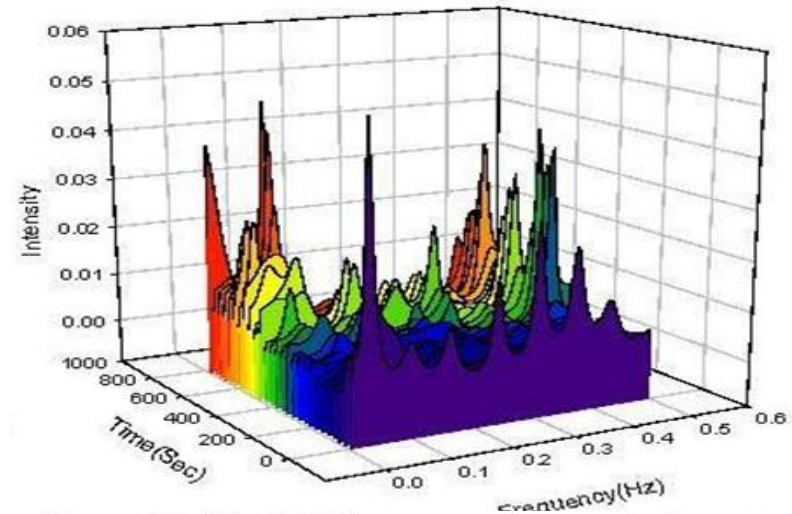
Result



Discussion



Frequency	Peak (Hz)	Power(ms2)	Power(%)	Power(mu)
V L F	0.0215	222	44.9	
LF	0.0469	184	37.2	67.5
HF	0.1504	89	17.9	32.5
LF/HF			2.079	



Frequency	Peak(Hz)	Power(ms2)	Power(%)	Power(mu)
V L F	0.0176	596	9.7	
LF	0.0488	1715	27.9	30.9
HF	0.2324	3862	62.4	69.1
LF/HF			0.447	

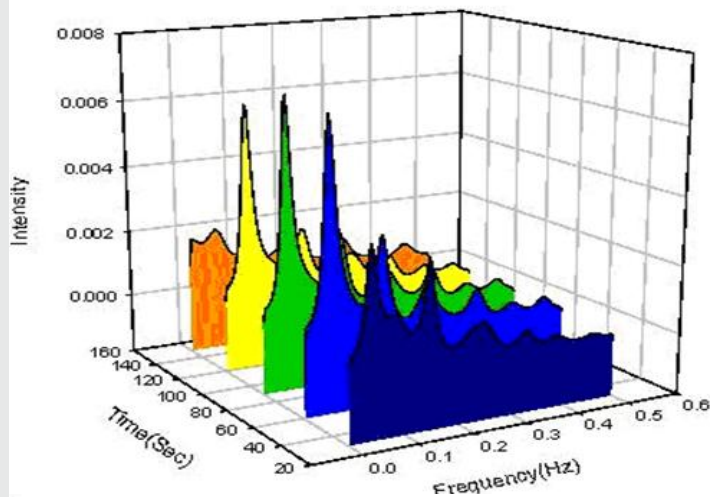
Normal Micturition 3D-Spectrogram

Introduction

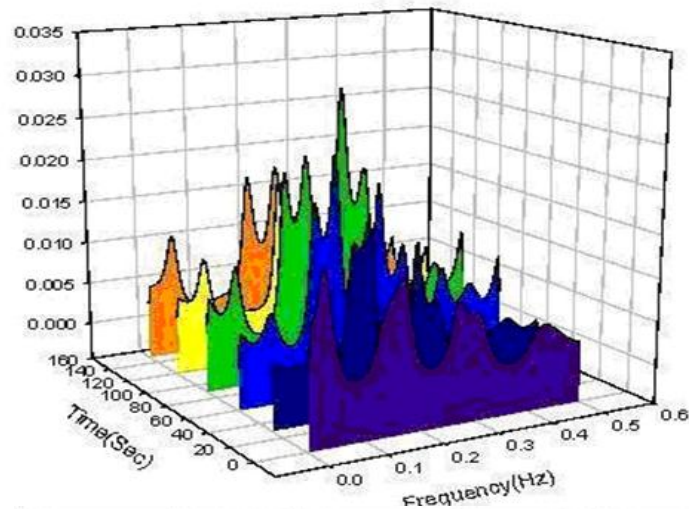
Method

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Frequency	Peak (Hz)	Power(ms2)	Power(%)	Power(nu)
V L F	0.0000	705	0.01	
LF	0.0703	452	58.8	58.81
HF	0.3184	316	41.1	41.19
LF/HF			1.43	



Frequency	Peak(Hz)	Power(ms2)	Power(%)	Power(nu)
V L F	0.0137	33	5.0	
LF	0.0586	87	13.1	13.6
HF	0.2695	541	81.9	86.2
LF/HF			0.160	

Micturition MSE Normal VS OAB

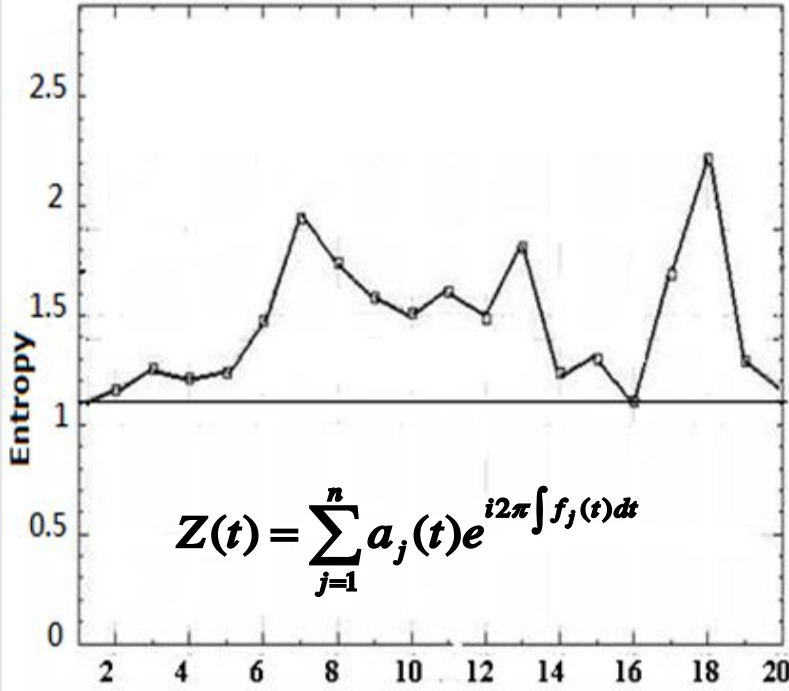
Introduction



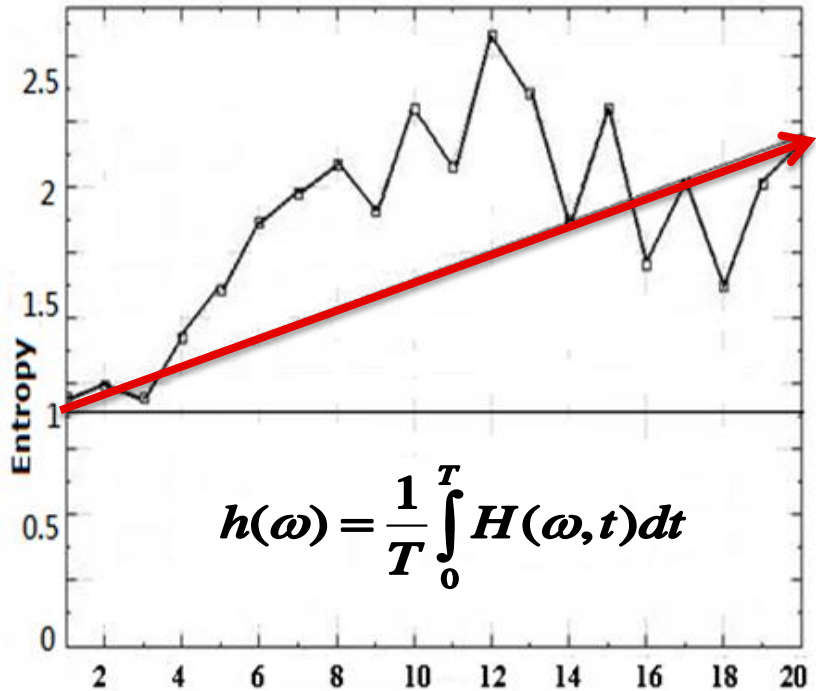
Result



Discussion



Complexity Scale
Normal Micturition

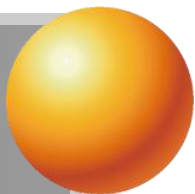


Complexity Scale
OAB

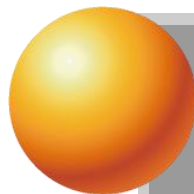
● Introduction 市場地位

目前，我們是領先

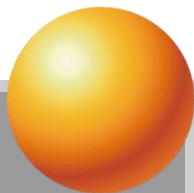
Collect
Data



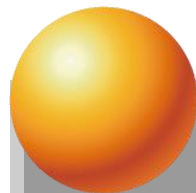
Analysis



Theorem



Application



學術討論
期待商用

Introduction

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Future Work

一個全球年需求365億美元,

感謝聆聽

Publication

Journal paper

- 1 [Liao Wen Chien](#) and Jaw F-S. “ Non-invasive Evaluation of Autonomic Nervous System Dysfunction in Female Patients with Overactive Bladder INTERNATIONAL JOURNAL OF GYNECOLOGY & OBSTETRICS Volme110,Issue1 Page12-17,July 2010 (SCI)
- 2 [Liao Wen Chien](#), and Jaw FS. “Noninvasive Impedance Analysis to Measure Human Urinary Bladder Volume” The Journal of Obstetrics and Gynaecology Research Accept Nov 2010 (SCI)
- 3 [Liao Wen Chien](#), and Jaw FS. “A Noninvasive Evaluation Analysis of Amniotic Fluid Embolism and Disseminated Intravascular Coagulopathy” The Journal of Maternal-Fetal & Neonatal Medicine Accepted: Dec 17 2010 (SCI)
- 4 [Liao Wen Chien](#) Yili Tseng, Yu-Rong Liang, Jaw F-S. “Portable Electrical Stimulator For Urinary Incontinence Biomedical Engineering: Applications, Basis and Communications Vol: 20, Issue: 1(2008) pp. 61-64 (SCI)

Paper

- 1 [Liao W-C](#), Jaw FS. Non-Invasive Impedance Measurement of Urinary Bladder Volume TUGA & TCS 2007; 12: P67
- 2 [Liao Wen Chien](#). Evaluation on the Autonomic Function of Normal Physiologic Storage and Micturition International Conference on Preventive Medicine Mar 25 2007
- 3 2009專利發明人1. 廖文劍 2. 高瑀絮 3. 趙福杉 4. 林啟萬
台灣大學專利案件發明名稱: [自律神經狀態的偵測裝置、分析裝置及分析方法]
美國發明專利申請案 已接獲官方專利公開通知台灣大學案號: 97工786US
德米國際專利商標事務所編號: O054-07002US 公開日: 2010/2/11 公開號: US-2010-0036267-A1 智產局收文文號 0973261253-0
- 4 [Liao W-C](#), Jaw FS. Non-Invasive Impedance Measurement Human Urinary Bladder Volume The 9th TCS & TUGA 2010; 12: P5
- 5 [Liao W-C](#), Jaw FS. Non-Invasive Evaluation of Autonomic Nervous System Dysfunction in Female Patients with Overactive BladderThe 9th TCS & TUGA 2010; 12: P4

$$\cos \frac{n2\pi t}{T} \Rightarrow \cos \omega_0 t \quad \sin \frac{n2\pi t}{T} \Rightarrow \sin \omega_n t \quad \omega_0 = \frac{2\pi}{T} \quad n \frac{2\pi}{T} = n\omega_0 = \omega_n$$

$$n=1, n\omega_0 = 1\omega_0 = \omega_1 \quad n=2, n\omega_0 = 2\omega_0 = \omega_2$$

$$e^{j\omega_n t} = \cos \omega_n t + j \sin \omega_n t \quad (e^{i\theta} = \cos \theta + i \sin \theta) \quad (e^{-i\theta} = \cos \theta - i \sin \theta)$$

$$\cos \omega_n t = \frac{e^{j\omega_n t} + e^{-j\omega_n t}}{2} \quad \sin \omega_n t = \frac{e^{j\omega_n t} - e^{-j\omega_n t}}{2j}$$

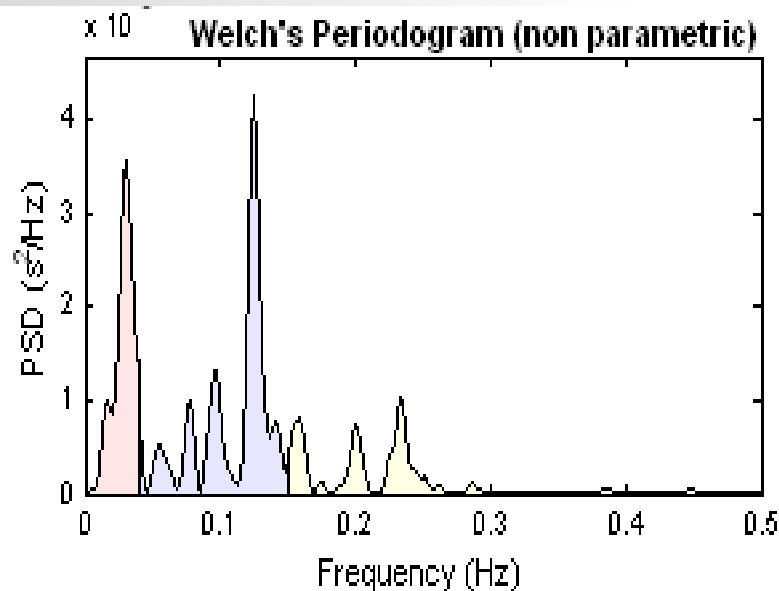
$$f(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos \omega_n t + b_n \sin \omega_n t] \quad f(t) = a_0 + \sum_{n=1}^{\infty} \left[a_n \left(\frac{e^{j\omega_n t} + e^{-j\omega_n t}}{2} \right) + b_n \left(\frac{e^{j\omega_n t} - e^{-j\omega_n t}}{2j} \right) \right]$$

$$c_0 = a_0 \quad c_n = \frac{a_n - jb_n}{2} \quad c_{-n} = \frac{a_n + jb_n}{2} \quad f(t) = C_0 + \sum_{n=1}^{\infty} [C_n e^{jn\omega_n t} + C_{-n} e^{-jn\omega_n t}] = \sum_{n=-\infty}^{\infty} C_n e^{j\omega_n t}$$

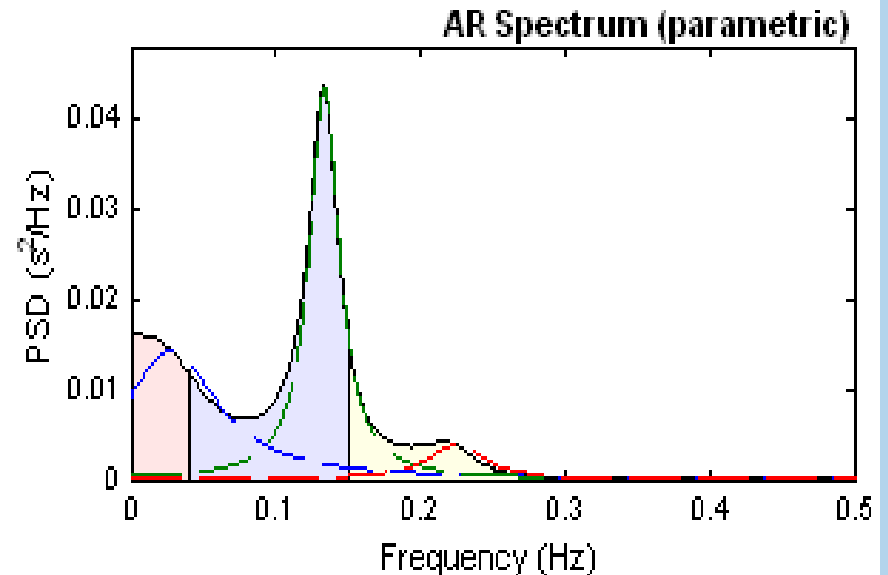
$$f(t) = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) e^{j\omega_n t} dt + C_0 e^{-j\omega_n t} + C_1 e^{-j(\omega_1 - \omega_n)t} + \bullet \bullet \bullet \bullet \bullet + C_n e^{-j(\omega_1 - \omega_n)t}$$

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

Power Spectrum Density



Frequency Band	Peak (Hz)	Power (ms^2)	Power (%)	Power (n.u.)
VLF	0.0313	51	29.7	
LF	0.1250	85	49.1	69.9
HF	0.2324	37	21.2	30.1
LF/HF			2.321	



Frequency Band	Peak (Hz)	Power (ms^2)	Power (%)	Power (n.u.)
VLF	0.0293	969	34.3	
LF	0.1328	1631	57.8	86.0
HF	0.2246	222	7.8	11.7
LF/HF			7.365	



Boltzaman

Total $u^* = u + u'$ if $u = u_r \rightarrow u = u_s \leftarrow$

$$\Omega^* = \Omega + \Omega' \leftarrow$$

$$\frac{p(u = u_s)}{p(u = u_r)} = \frac{p(u_s)}{p(u_r)} = \frac{\Omega^*(u = u_s)}{\Omega^*(u = u_r)} = \frac{\Omega(u_s)\Omega'(u^* - u_s)}{\Omega(u_r)\Omega'(u^* - u_r)} = G \bullet R$$

$$\frac{p(u_s)}{p(u_r)} = GR \quad G = \frac{\Omega(u_s)}{\Omega(u_r)} \quad R = \frac{\Omega'(u^* - u_s)}{\Omega'(u^* - u_r)} \leftarrow$$

$$\frac{1}{\Omega} \left(\frac{d\Omega'}{du'} \right) = \frac{1}{K_B T'} \quad \frac{d\Omega'}{du'} = \left(\frac{1}{K_B T'} \right) \Omega' \leftarrow$$

$$\Omega'(u') = const \times e^{\frac{u'}{K_B T'}}$$

$$\frac{\Omega'(u^* - u_s)}{\Omega'(u^* - u_r)} = R = \frac{const \times e^{\frac{u^* - u_s}{K_B T'}}}{const \times e^{\frac{u^* - u_r}{K_B T'}}} = e^{\frac{-(u_s - u_r)}{K_B T'}} = e^{\frac{-(u_s - u_r)}{K_B T'}} \leftarrow$$

$$\frac{p(u_s)}{p(u_r)} = G \bullet R = Ge^{\frac{-(u_s - u_r)}{K_B T'}} = \left[\frac{\Omega(u_s)}{\Omega(u_r)} \right] e^{\frac{-(u_s - u_r)}{K_B T'}}$$



Electrical Stimulation

Boltzman factor ↵

Total $u^* = u + u'$ if $u = u_y \rightarrow u = u_s$ ↵

$$\Omega^* = \Omega + \Omega' \quad \leftarrow$$

$$\frac{p(u = u_s)}{p(u = u_y)} = \frac{p(u_s)}{p(u_y)} = \frac{\Omega^*(u = u_s)}{\Omega^*(u = u_y)} = \frac{\Omega(u_s)\Omega'(u^* - u_s)}{\Omega(u_y)\Omega'(u^* - u_y)} \quad \frac{P_2}{P_1} = \frac{C_2}{C_1} = \frac{\Omega(2)}{\Omega(1)} e^{\frac{-(u_2 - u_1)}{K_B T}} \quad \frac{\Omega(2)}{\Omega(1)} = \frac{\alpha \Delta x \Delta y \Delta z}{\alpha \Delta x \Delta y \Delta z} = 1$$

$$\frac{p(u_s)}{p(u_y)} = GR \quad G = \frac{\Omega(u_s)}{\Omega(u_y)} \quad R = \frac{\Omega'(u^* - u_s)}{\Omega'(u^* - u_y)} \quad u_2 - u_1 = E_k(2) - E_k(1) + Ze(V_2 - V_1)$$

$$\frac{1}{\Omega} \left(\frac{d\Omega'}{du'} \right) = \frac{1}{K_B T'} \quad \frac{d\Omega'}{du'} = \left(\frac{1}{K_B T'} \right) \Omega \quad \frac{C_2}{C_1} = e^{\frac{-Ze(V_2 - V_1)}{K_B T}}$$

$$\Omega'(u') = \text{const} \times e^{\frac{u'}{K_B T}}$$

$$\frac{\Omega'(u^* - u_s)}{\Omega'(u^* - u_y)} = R = \frac{\text{const} \times e^{\frac{u^* - u_s}{K_B T}}}{\text{const} \times e^{\frac{u^* - u_y}{K_B T}}} = e^{\frac{-(u_s - u_y)}{K_B T}} = e^{\frac{-(u - u_y)}{K_B T}} \quad \ln\left(\frac{C_2}{C_1}\right) = \frac{-Ze}{K_B T} (V_2 - V_1) \Rightarrow V_2 - V_1 = \frac{K_B T}{Ze} \ln\left(\frac{C_2}{C_1}\right) \quad \leftarrow$$

$$\frac{p(u_s)}{p(u_y)} = G \cdot R = Ge^{\frac{-(u - u_y)}{K_B T}} = \left[\frac{\Omega(u_s)}{\Omega(u_y)} \right] e^{\frac{-(u - u_y)}{K_B T}}$$

Nernst Equation ↵

$$\frac{C_2}{C_1} = \frac{P(2)}{P(1)} \quad u = E_k + E_p \quad E_p = ZeV \quad \leftarrow$$

$$\frac{P_2}{P_1} = \frac{C_2}{C_1} = \frac{\Omega(2)}{\Omega(1)} e^{\frac{-(u_2 - u_1)}{K_B T}} \quad \frac{\Omega(2)}{\Omega(1)} = \frac{\alpha \Delta x \Delta y \Delta z}{\alpha \Delta x \Delta y \Delta z} = 1$$

$$u_2 - u_1 = E_k(2) - E_k(1) + Ze(V_2 - V_1)$$

$$\frac{C_2}{C_1} = e^{\frac{-Ze(V_2 - V_1)}{K_B T}}$$

$$\ln\left(\frac{C_2}{C_1}\right) = \frac{-Ze}{K_B T} (V_2 - V_1) \Rightarrow V_2 - V_1 = \frac{K_B T}{Ze} \ln\left(\frac{C_2}{C_1}\right) \quad \leftarrow$$

$$N_A \cdot K_B = R = 8.31451 \text{ J} \cdot \text{mol}^{-1} \cdot \text{K}^{-1}$$

$$N_A \cdot e = F = 96485.31 \text{ C} \cdot \text{mol}^{-1}$$

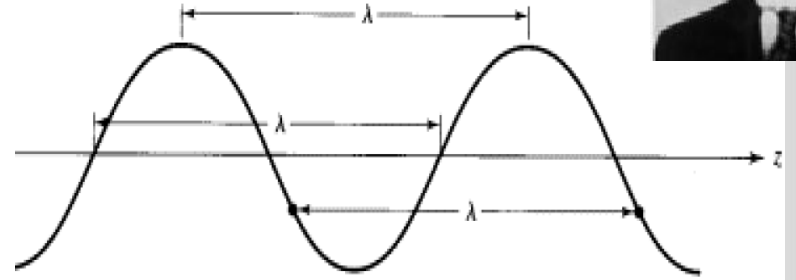
$$\frac{K_B T}{Ze} = \frac{RT}{ZF} \Rightarrow V_2 - V_1 = \frac{RT}{ZF} \ln\left(\frac{C_1}{C_2}\right)$$

de Broglie, *Recherches sur la théorie des quanta* (Researches on the quantum theory), Thesis (Paris), 1924; L. de Broglie, *Ann. Phys.* (Paris)



de Broglie Hypothesis

Lightwave \rightleftharpoons ? Photon
MatterWave \leftarrow ? Particle



$$(1) E = h\nu$$

(h: Planck's Constant
ν: Frequency)

$$(2) p = \frac{h}{\lambda}$$

$$(3) \lambda = \frac{h}{p}$$

(h: Planck's constant
= 6.626×10^{-34} Js)

$$v = \frac{\lambda}{T} = \lambda \nu \quad \frac{1}{T} = \nu \quad \nu = \frac{c}{\lambda}$$

$$E = mc^2 \quad E = h\nu = h \frac{c}{\lambda}$$

$$mcc = h \frac{c}{\lambda} \quad mc = \frac{h}{\lambda}$$

$$p = \frac{h}{\lambda} \quad mc = p$$

$$\lambda = \frac{h}{p}$$

Quantum Theory

de Broglie Equation

$$\lambda = \frac{h}{p} \quad \lambda = \frac{h}{p} = \frac{h}{\gamma m v} = \frac{h}{m v} \sqrt{1 - \frac{v^2}{c^2}}$$

Schrödinger Equation (波動力學)

$$H(t)\psi(t) \rangle = i\hbar \frac{\partial}{\partial t} \psi(t) \rangle$$

Heisenberg uncertainty principle

$$\Delta E \cdot \Delta T \geq h \quad \Delta E \Delta t \geq \frac{\hbar}{2}$$

Heisenberg uncertain Principle2

$$\lambda_q \lambda_l = \left(\frac{h}{mv}\right)(CT_l) \quad C = \frac{\lambda_l}{T_l} \quad \lambda_q \lambda_l = \frac{h}{p}(CT_l)$$

$$P \lambda_q \left(\frac{\lambda_l}{C}\right) = h(T_l) \quad P \left(\frac{\lambda_q}{T_l}\right) \left(\frac{\lambda_l}{C}\right) = h \quad T_l = \frac{\lambda_l}{C}$$

$$\Delta v = \frac{\lambda_q}{T_l} \quad T_l = \frac{\lambda_l}{C} \quad P = mv$$

$$P \Delta v T_l = h$$

$$(\because \Delta P = m \Delta v)$$

$$\frac{P m \Delta v}{m} T_l = h$$

$$\frac{P \Delta P}{m} T_l = h \quad P = mv \quad E = \frac{1}{2} m v^2 \quad E = \frac{1 m^2 v^2}{2m}$$

$$E = \frac{P^2}{2m} \quad \Delta E = \frac{2 P \Delta P}{2m} \quad \Delta E = \frac{P \Delta P}{m} \quad \Delta T = T_l$$

$$\Delta E \Delta t = h$$

indeterminacy relation

$$\Delta f \Delta t = h$$