國立台灣師範大學機電科技研究所

Multi-Scale Entropy 在機械震動訊號之應用

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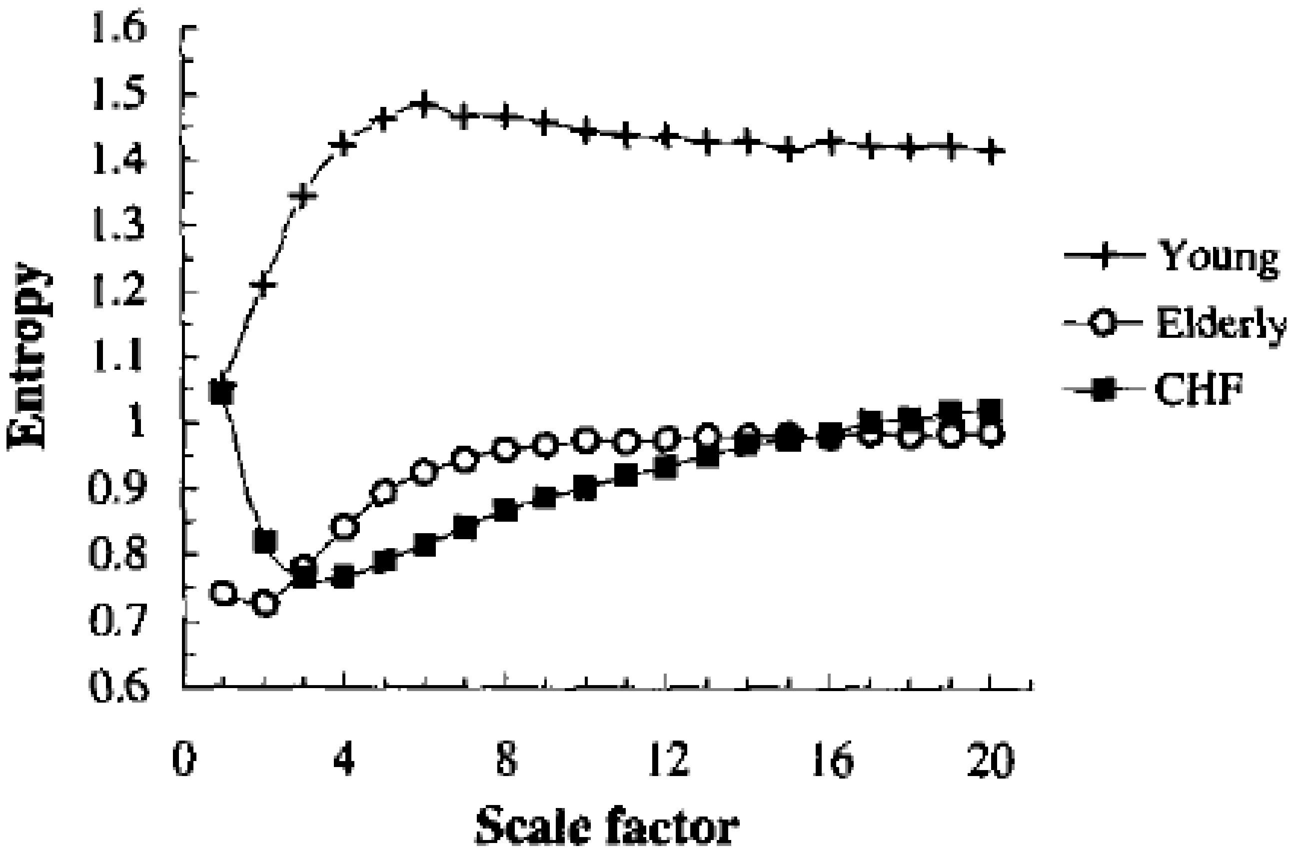
Outline

- Measure of Complexity
- Sample Entropy
- Multi Scale Entropy
- Analysis of vibration signal based on MSE
- Fault detection of rotary machine based on MSE

MSE analysis of interbeat interval time-series



Measure of Complexity

- Information theory
  - Shannon's entropy
  - Approximate entropy (Pincus (1991))
  - Sample entropy (Richman and Moorman (2000))
  - Fourier entropy
  - Wavelet entropy (Rosso et.al. (2003))
  - Renyi Entropy (Gonzalez, et.al. 2000)
  - Higher Order methods. (Gu, et.al. 2004)

Measure of Complexity

- Chaos-based estimates of complexity
  - Lyapunov exponent
  - permutation entropy (Bandt and Pompe (2002))
- Komologrov estimates (algorithmic complexity)
  - Lempel-Ziv (Ziv and Lempel 1978, Evans, et.al. 2002)

Sample Entropy

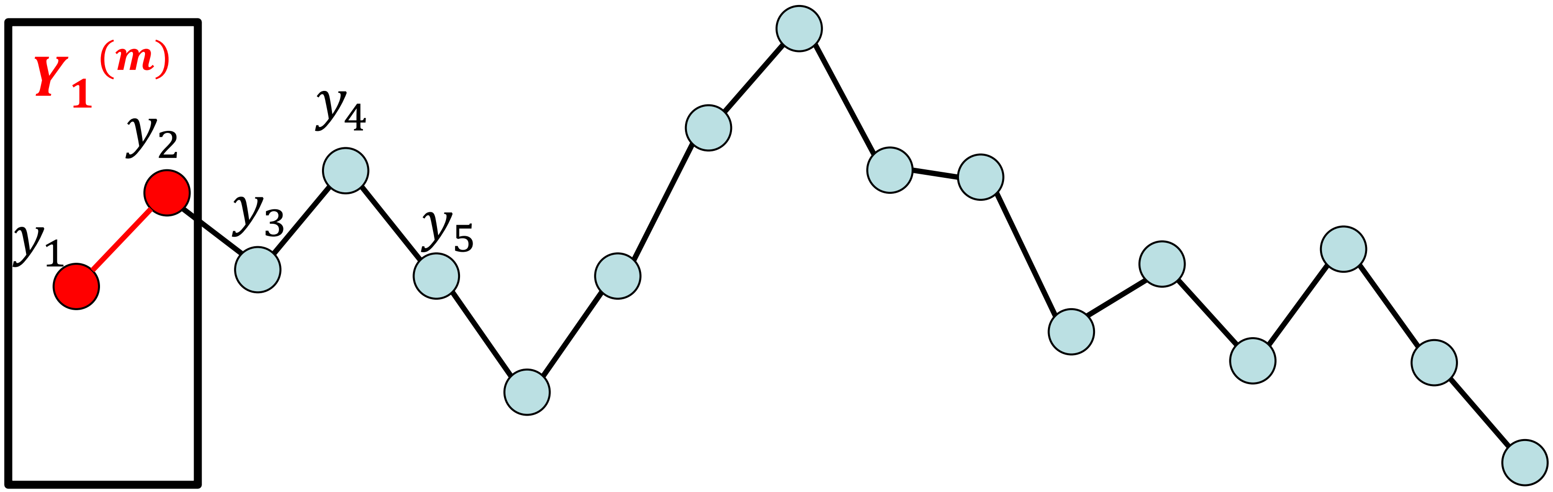
- Sample Entropy is the negative natural logarithm of an estimate of the conditional probability that subseries (epochs) of length *m* that match pointwise within a tolerance ϵ also match at the next point

$$\mathrm{SampEn}(\mathrm{m}, \epsilon) = -\log\left(\frac{\phi(m+1,\epsilon)}{\phi(m,\epsilon)}\right)$$

$\phi(m, \epsilon)$ is define to be the number of matches of length m within a tolerance ϵ

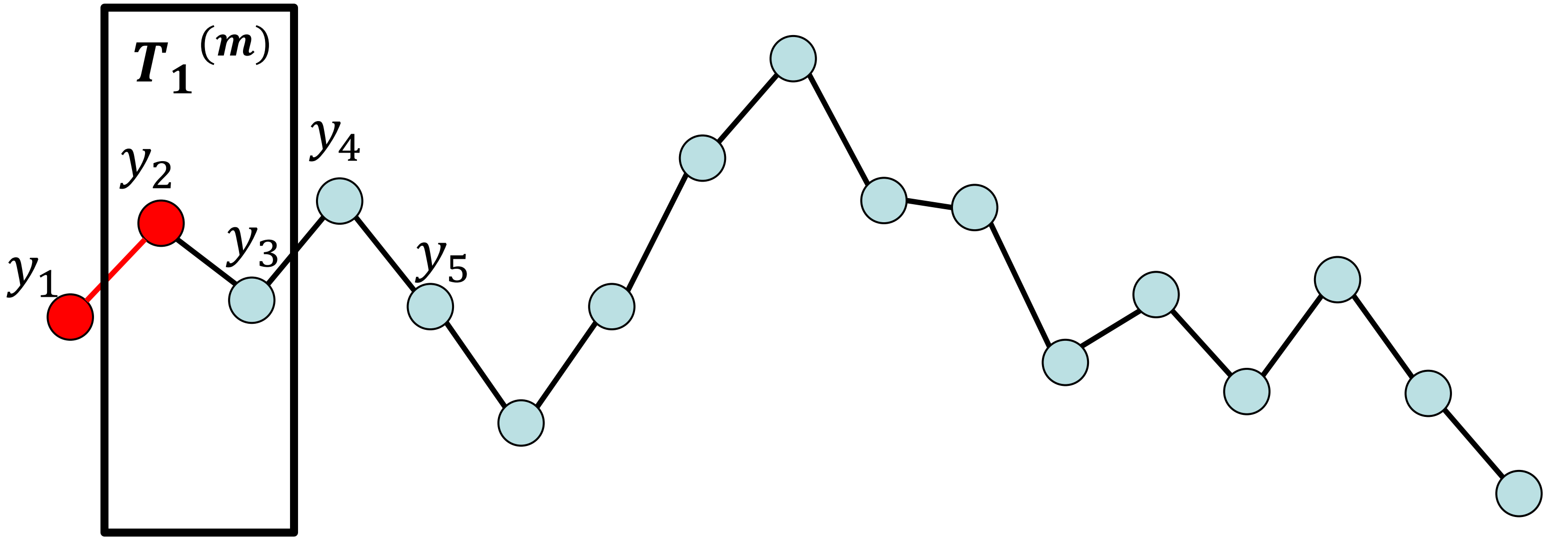
Sample Entropy (Example)

- $m = 2$



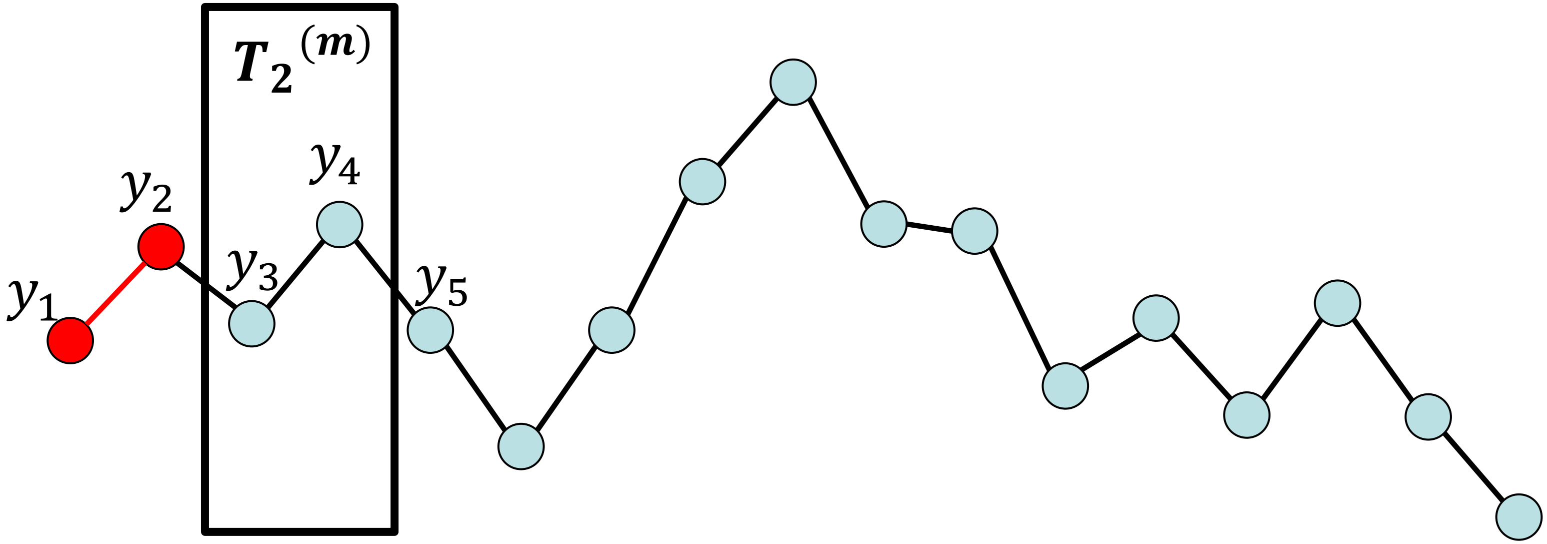
Sample Entropy (Example)

- $C(m, \epsilon) = 0$



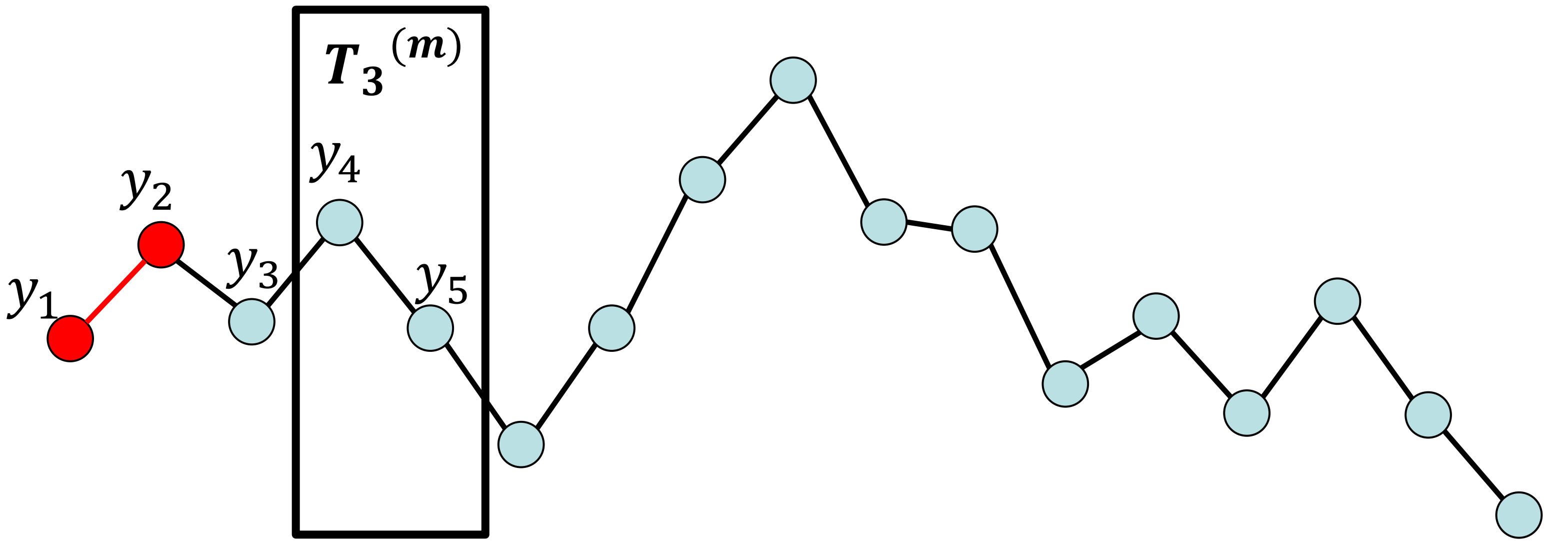
Sample Entropy (Example)

- $C(m,\epsilon) = 1$



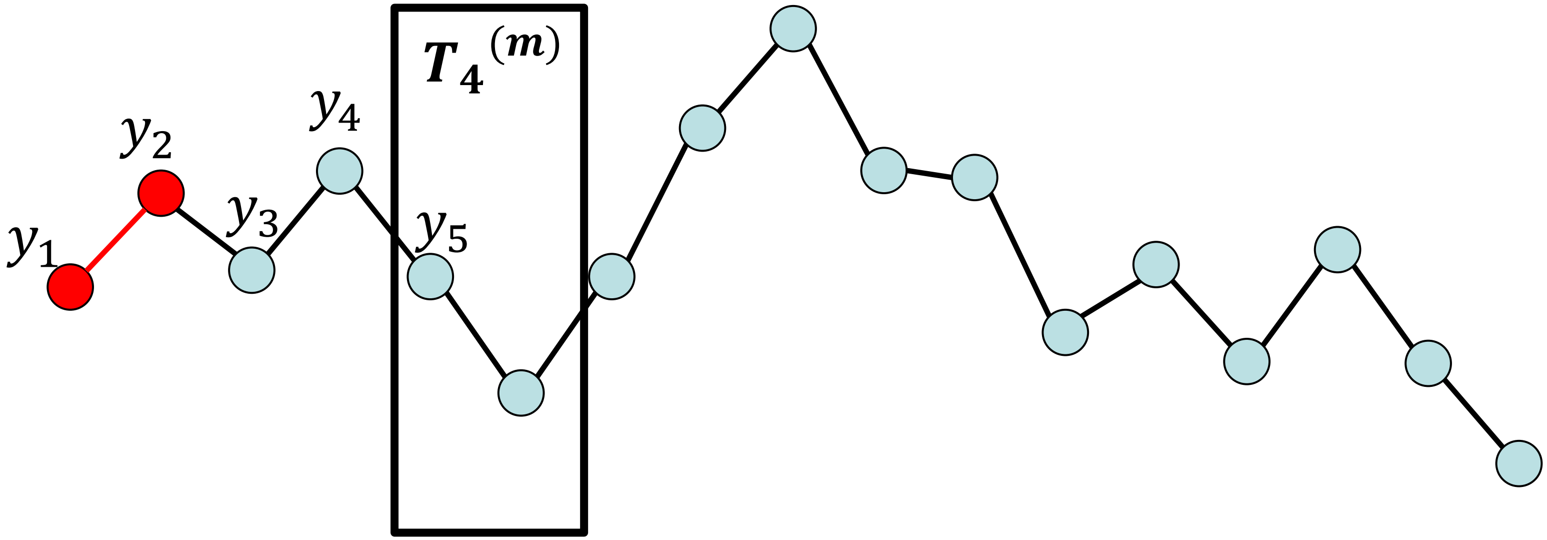
Sample Entropy (Example)

- $C(m, \epsilon) = 1$



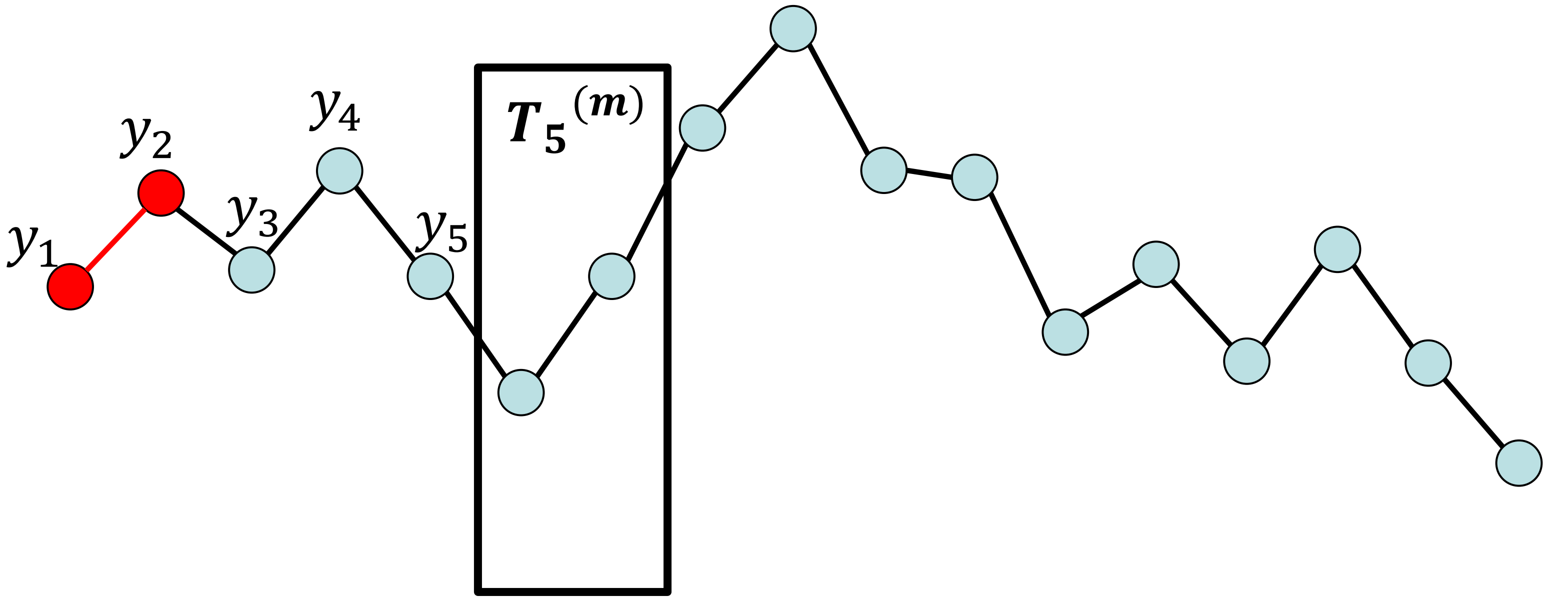
Sample Entropy (Example)

- $C(m,\epsilon) = 1$



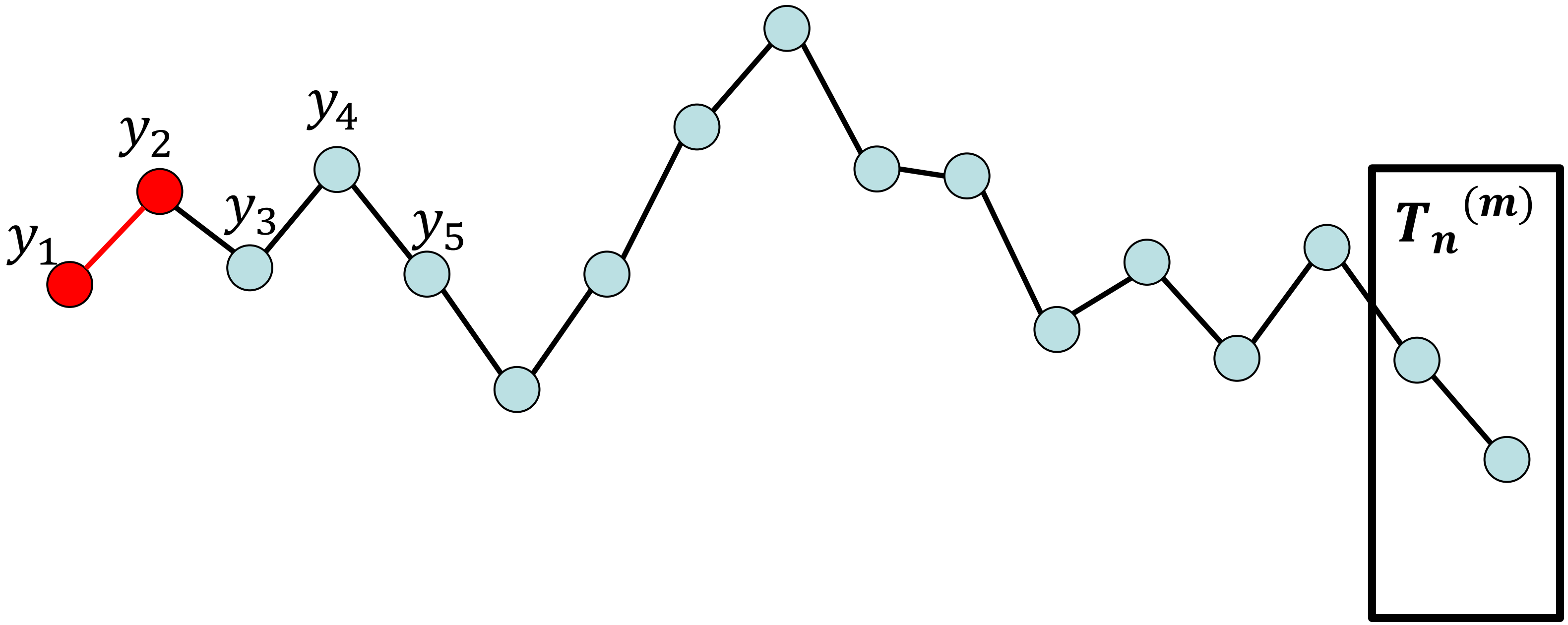
Sample Entropy (Example)

- $C(m, \epsilon) = 2$



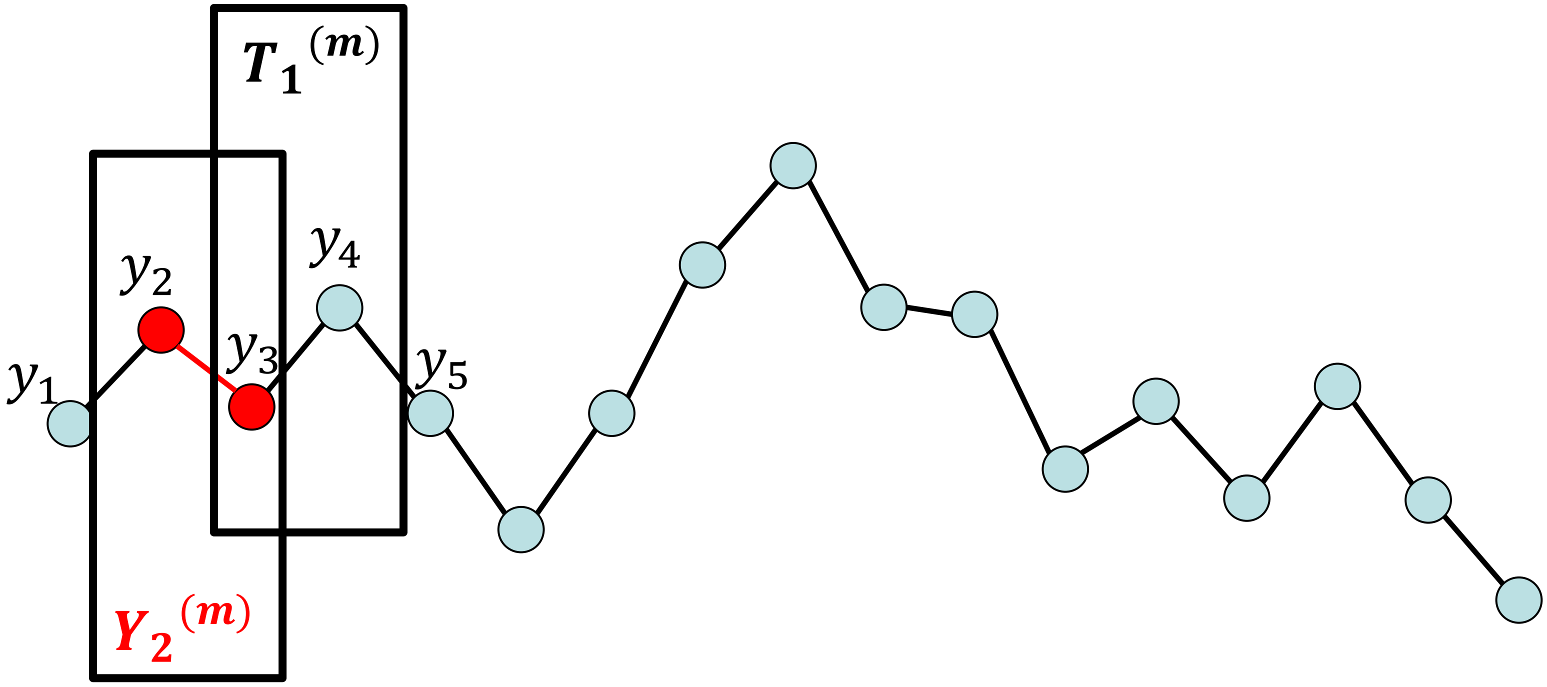
Sample Entropy (Example)

- $C(m, \epsilon) = 7$



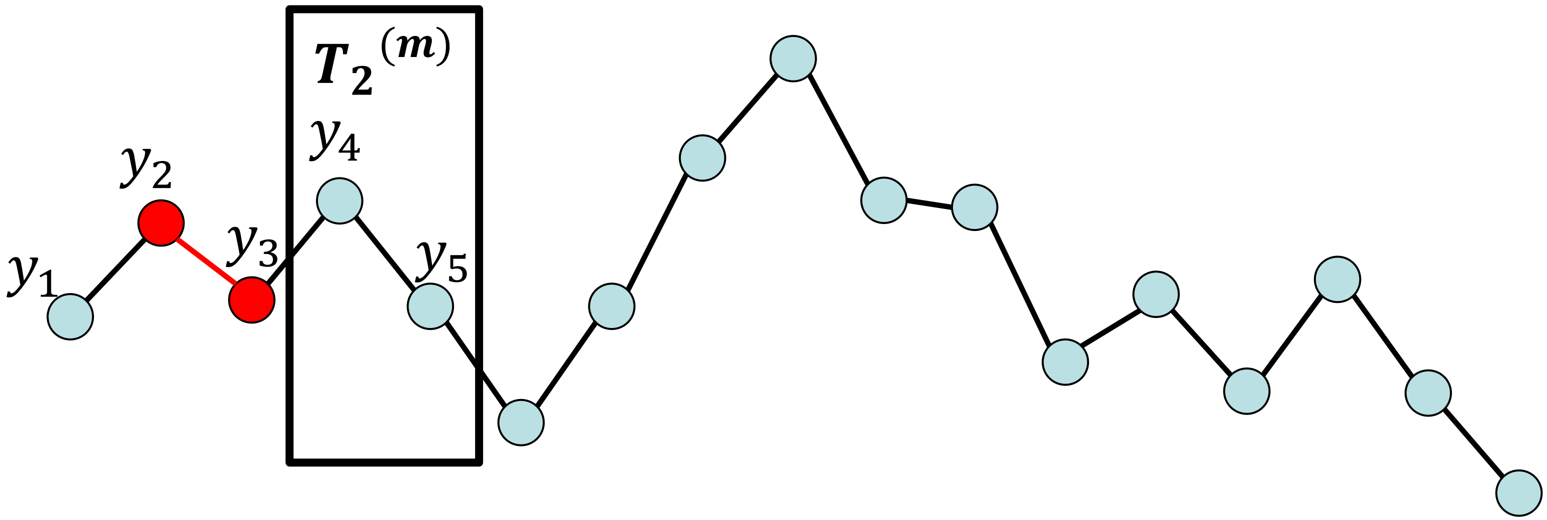
Sample Entropy (Example)

- $C(m, \epsilon) = 7.$



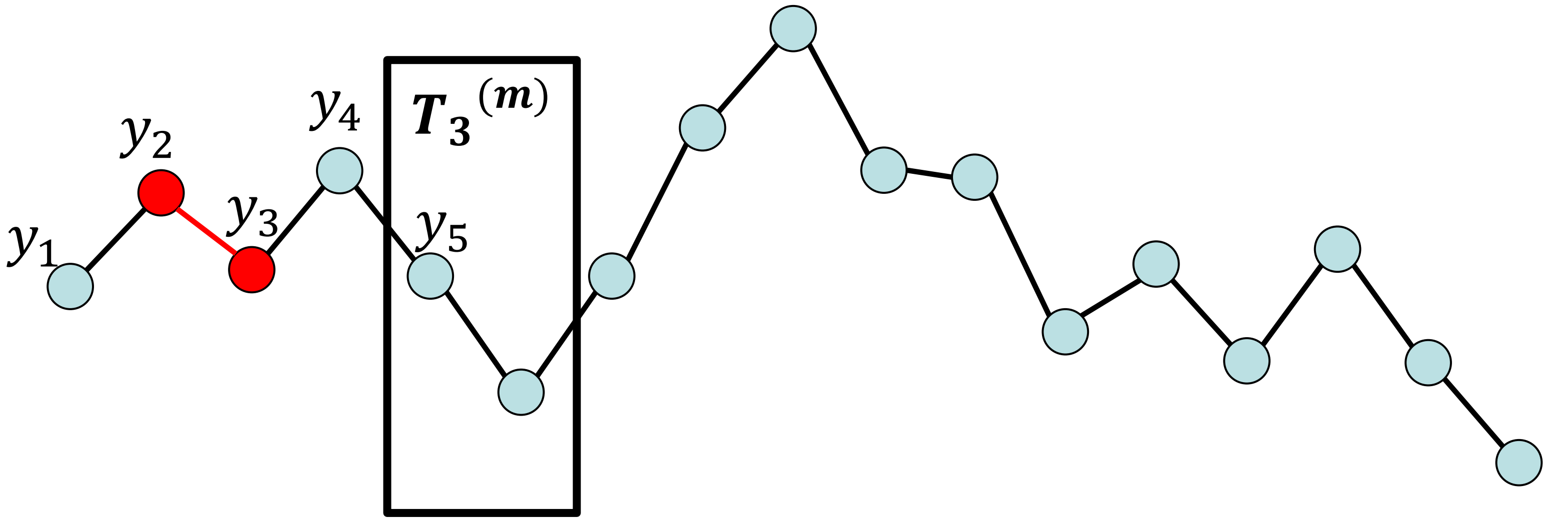
Sample Entropy (Example)

- $C(m,\epsilon) = 8$



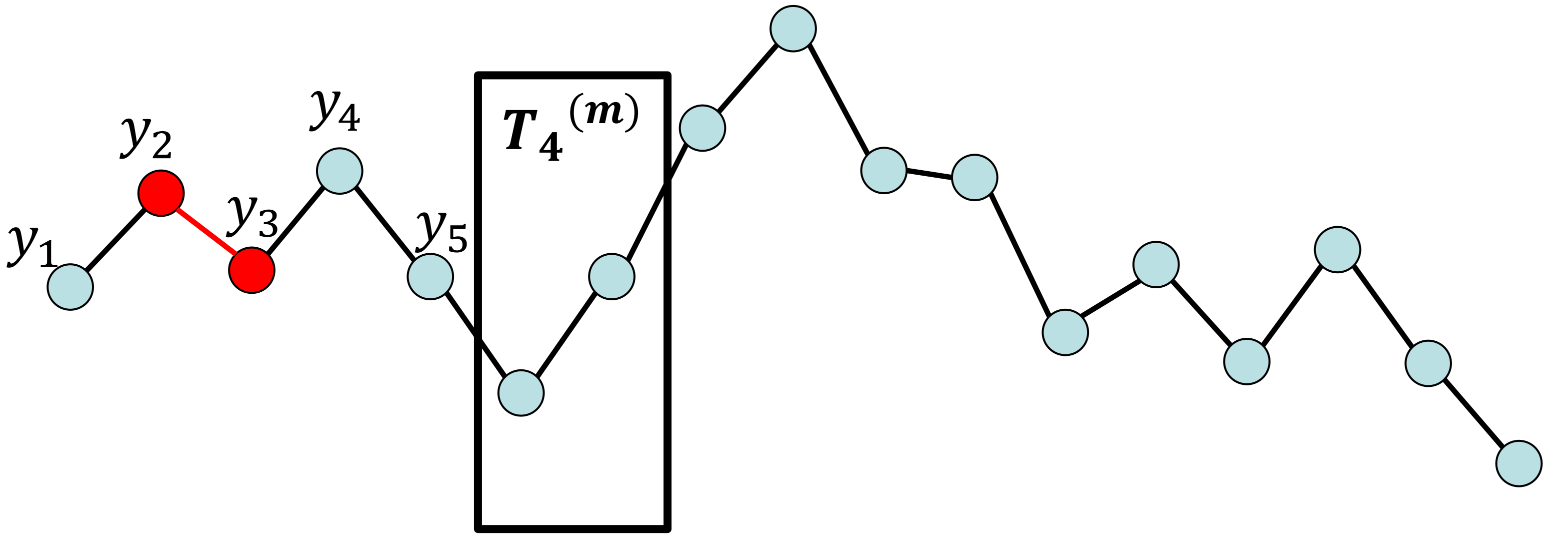
Sample Entropy (Example)

- $C(m,\epsilon) = 9$



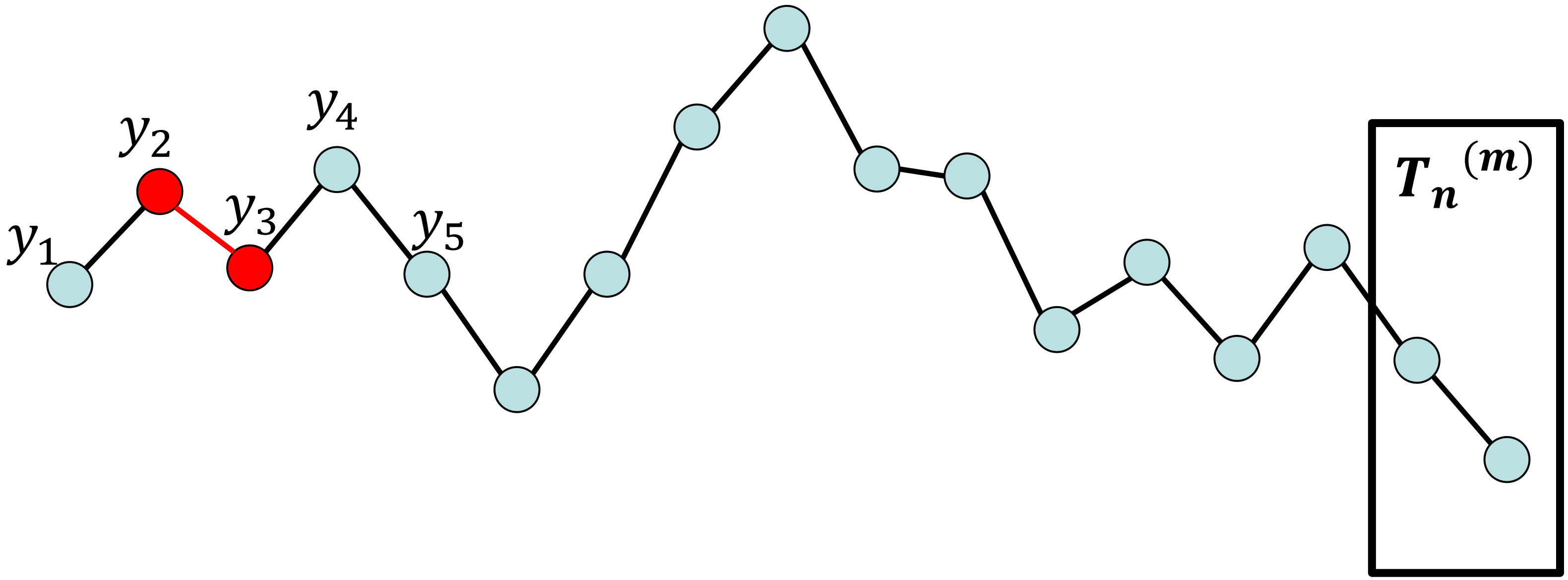
Sample Entropy (Example)

- $C(m, \epsilon) = 9$



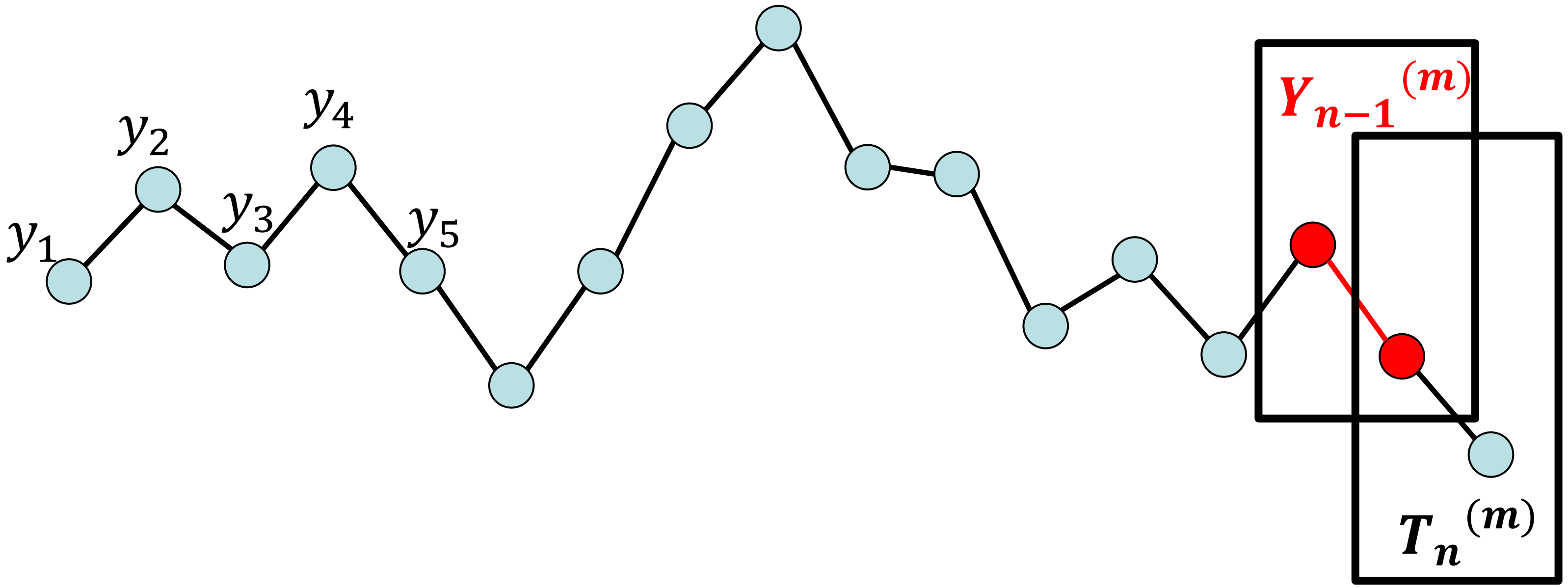
Sample Entropy (Example)

- $C(m, \epsilon) = 12$



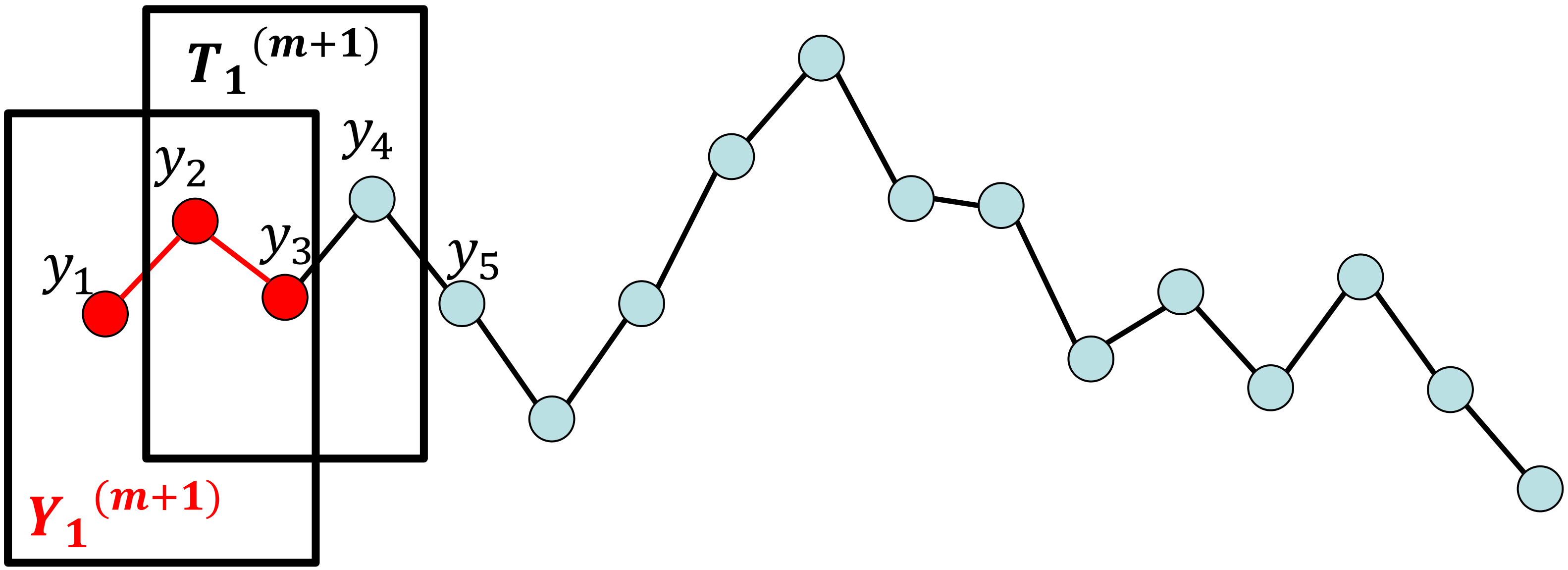
Sample Entropy (Example)

- $C(m,\epsilon) = 40$



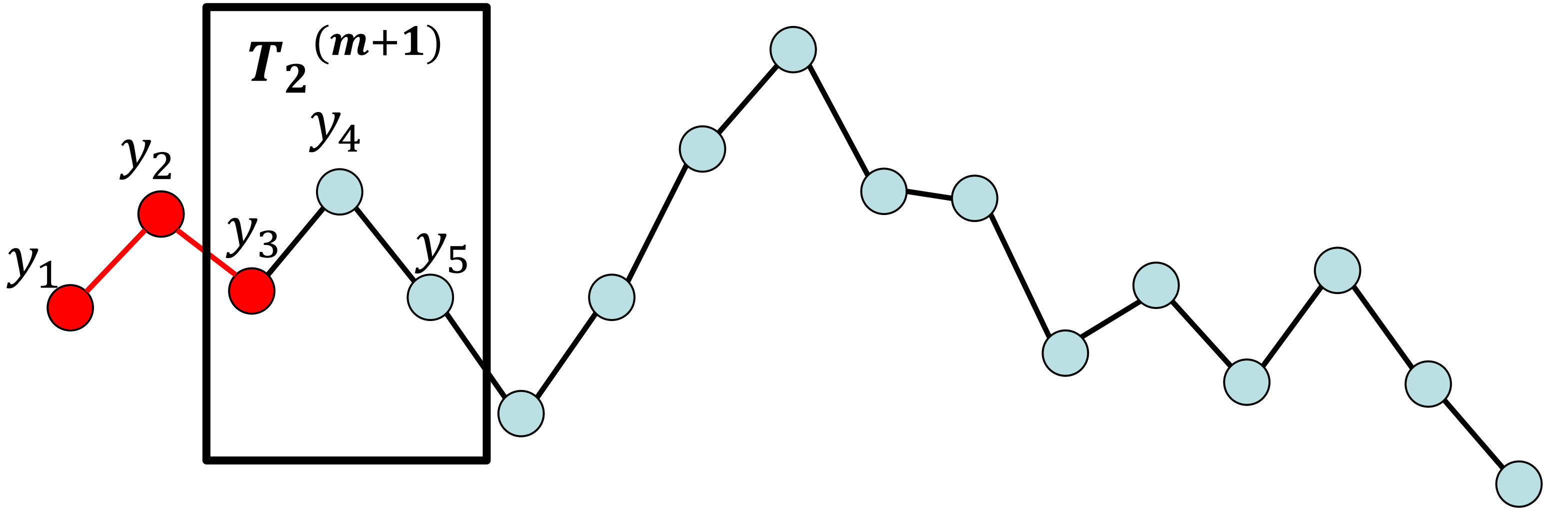
Sample Entropy (Example)

- $C(m+1, \epsilon) = 0$



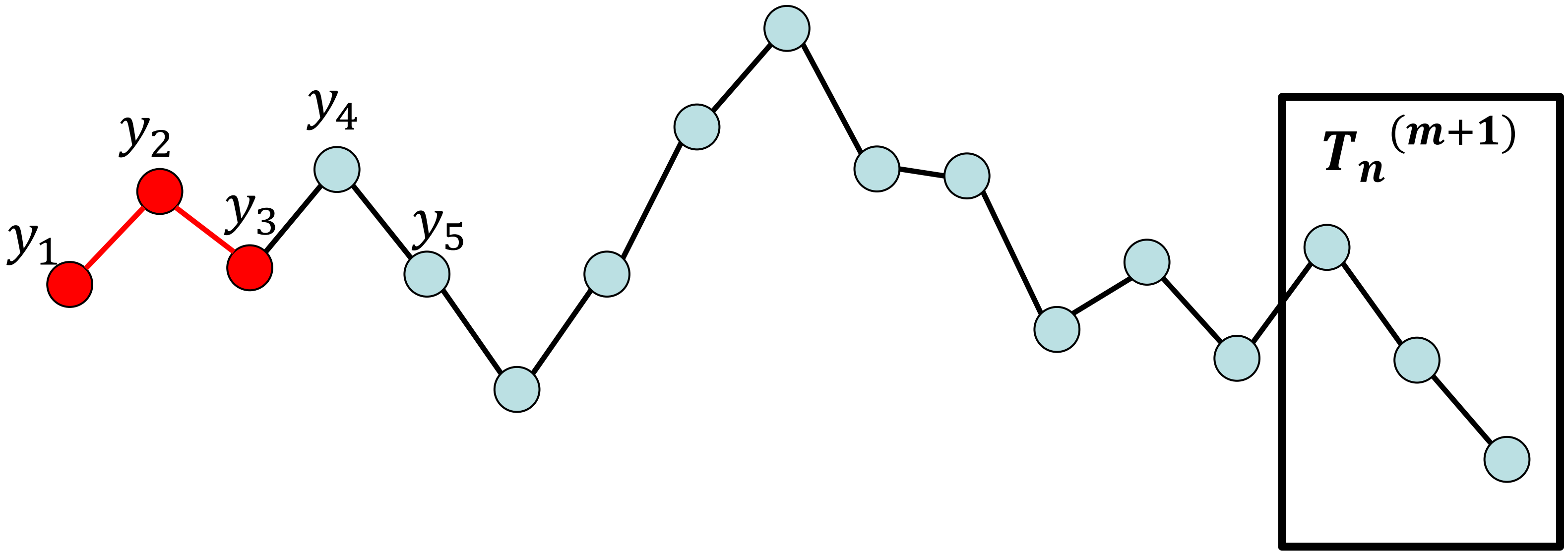
Sample Entropy (Example)

- $C(m+1, \epsilon) = 0$



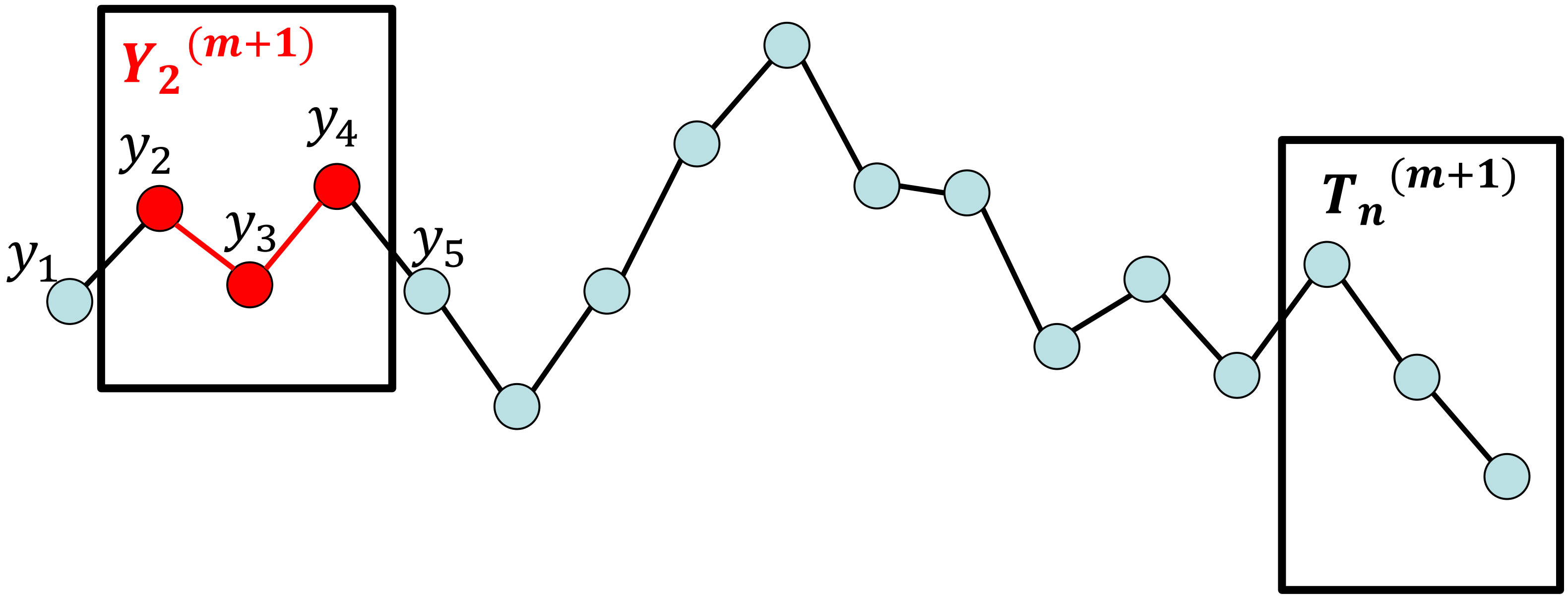
Sample Entropy (Example)

- $C(m+1,\epsilon) = 3$



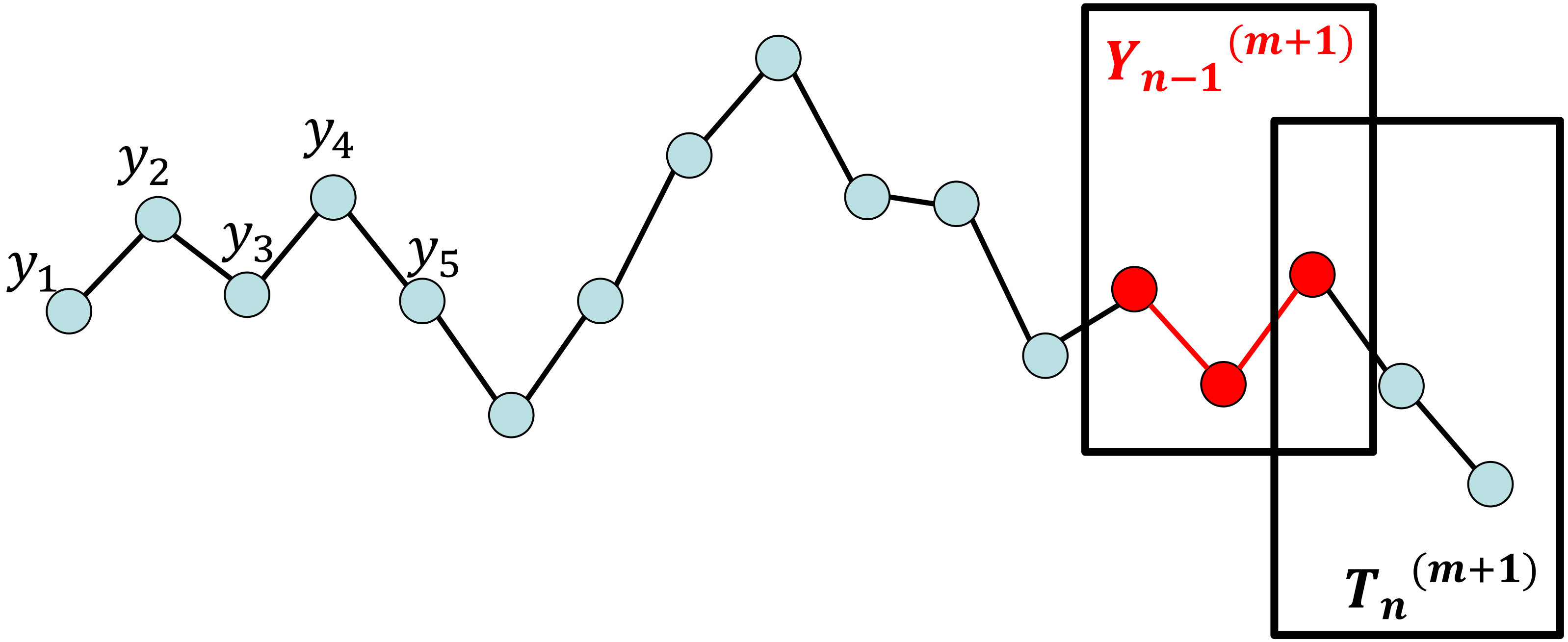
Sample Entropy (Example)

- $C(m+1,\epsilon) = 5$



Sample Entropy (Example)

- $C(m+1, \epsilon) = 11$



Sample Entropy (Example)

- $C(m,\epsilon) = 40$
- $C(m+1,\epsilon) = 11$
- $\mathrm{SampEn}(\mathrm{m},\epsilon) = -\log\left(\frac{C(m+1,\epsilon)}{C(m,\epsilon)}\right) = 1.291$

Sample Entropy

- Algorithm

Input: Data x of length N, patter length m, tolerance r.
Procedure:

$$X = \begin{bmatrix} x(1) & x(2) & \cdots & x(m) \\ x(2) & x(3) & \cdots & x(m+1) \\ \vdots & \vdots & & \vdots \\ x(N-m) & x(N-m+1) & & x(N-1) \end{bmatrix};$$

For $i = 1$ to $N-m$
For j= 1 to $N-m\ (j \neq i)$

$$d(i,j) = \max\left(\left|X(i,1)-X(j,1)\right|, \left|X(i,2)-X(j,2)\right|, \cdots, \left|X(i,i+m-1)-X(j,j+m-1)\right|\right);$$

$$D(i,j) = H\left(d(i,j)\right);$$

End

$$B(i) = \frac{1}{N-m-1}\sum_{j=1}^{N-m+1} D(i,j);$$

End

$$\phi_m = \frac{1}{N-m}\sum_{i=1}^{N-m} B(i);$$

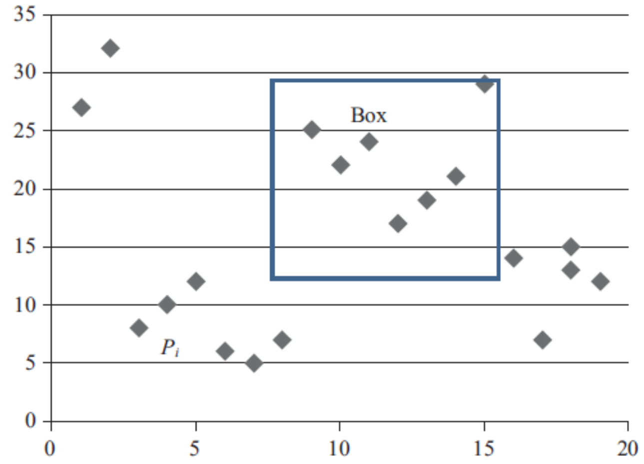
Similar computations are performed for pattern length m + resulting in ϕ_{m+1}, with

$$X = \begin{bmatrix} x(1) & x(2) & \cdots & x(m+1) \\ x(2) & x(3) & \cdots & x(m+2) \\ \vdots & \vdots & & \vdots \\ x(N-m) & x(N-m+1) & & x(N) \end{bmatrix};$$

Output: $\mathrm{SampEn}(x,m,r) = \ln\phi_m - \ln\phi_{m+1}$.

Sample Entropy

- Algorithm




```
for i=1:N {
 for j=i+1:N {
       if (|f_i − f_j) < ε and |f_{i+1} − f_{j+1}| < ε)
  n_n = n_n + 1 /* compute numerator in (4) */
        if (| f_{i+2} − f_{j+2}| < ε) {
            n_d = n_d + 1; /* compute denominator in (4) */
        } // if
      } // if
   } // j
 } // i
 Compute entropy [scale] = -log (n_n / n_d); // from (3)
```

Sample Entropy Fastest algorithm

- Brute force method [Richardman. 2000]
  - $O(N^2)$
- k-d Tree [Y.H. Pan 2011]
  - $O(N^{5/3})$
  - The algorithm have been implemented into visual signal

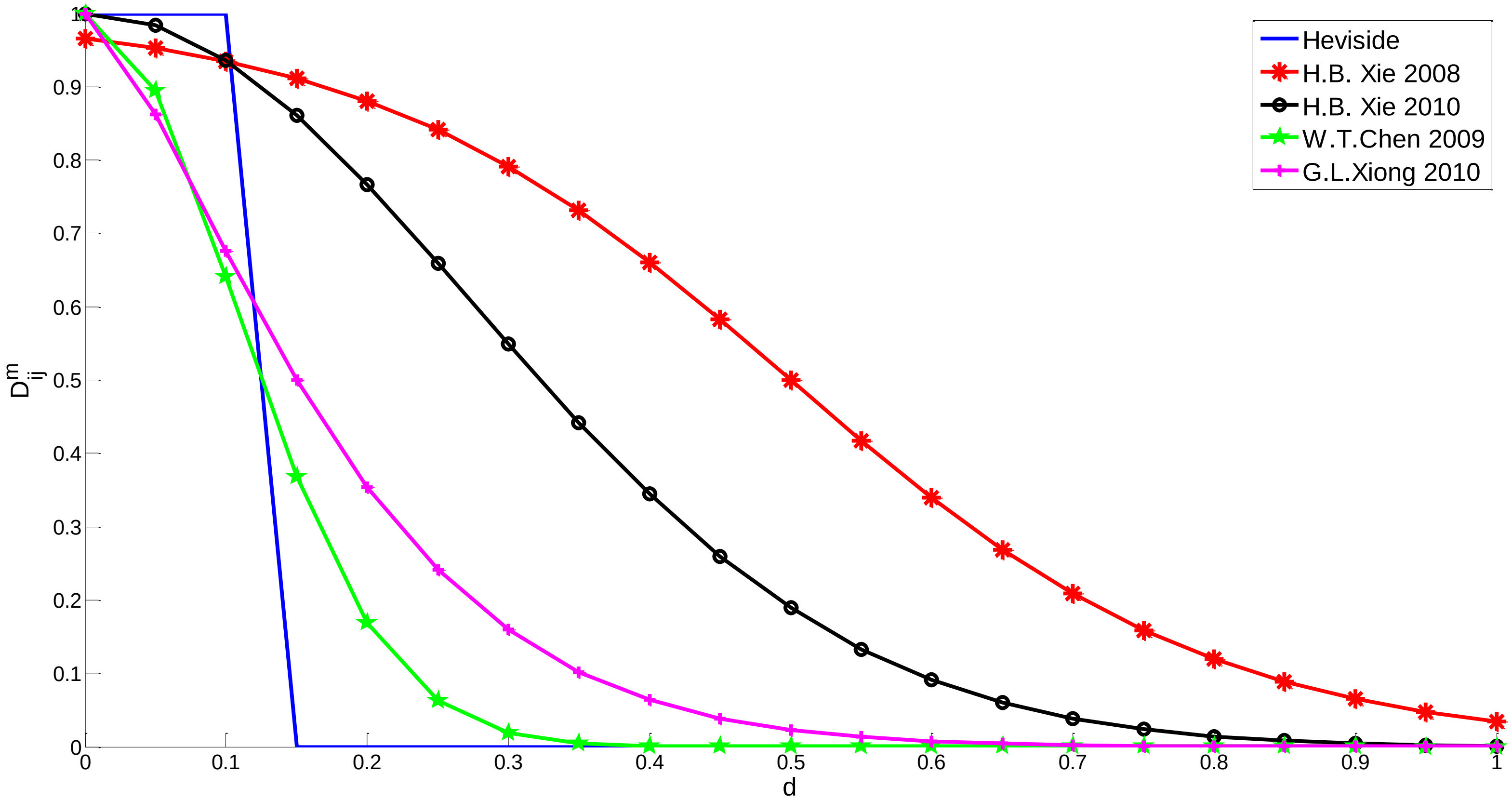
Sample Entropy
Different distant function

$$d = d_{ij}^{(m)} = d[Y_i^{(m)}, T_j^{(m)}] = \|(Y_i^{(m)}, T_j^{(m)})\|_\infty$$

- Heviside
  - $\begin{cases} D_{ij}^{(m)} = 1 & if\ d \le \epsilon \\ D_{ij}^{(m)} = 0 & if\ d > \epsilon \end{cases}$
- Modify
  - $D_{ij}^{(m)} = \frac{1}{1+\exp((d-0.5)/\epsilon)}$ [H.B. Xie, 2008]
  - $D_{ij}^{(m)} = \exp(-d^2/\epsilon)$ [H.B. Xie, 2010]
- Fuzzy
  - $D_{ij}^{(m)} = \exp(-(d/\epsilon)^n)$ [W.T. Chen, 2009]
  - $D_{ij}^{(m)} = \exp(-\frac{d^{\ln(\ln 2^C)/\ln \epsilon}}{c})$ [G.L. Xiong, 2010]

Sample Entropy
Different distant function

- $\epsilon = 0.15$



Sample Entropy
Different distant function

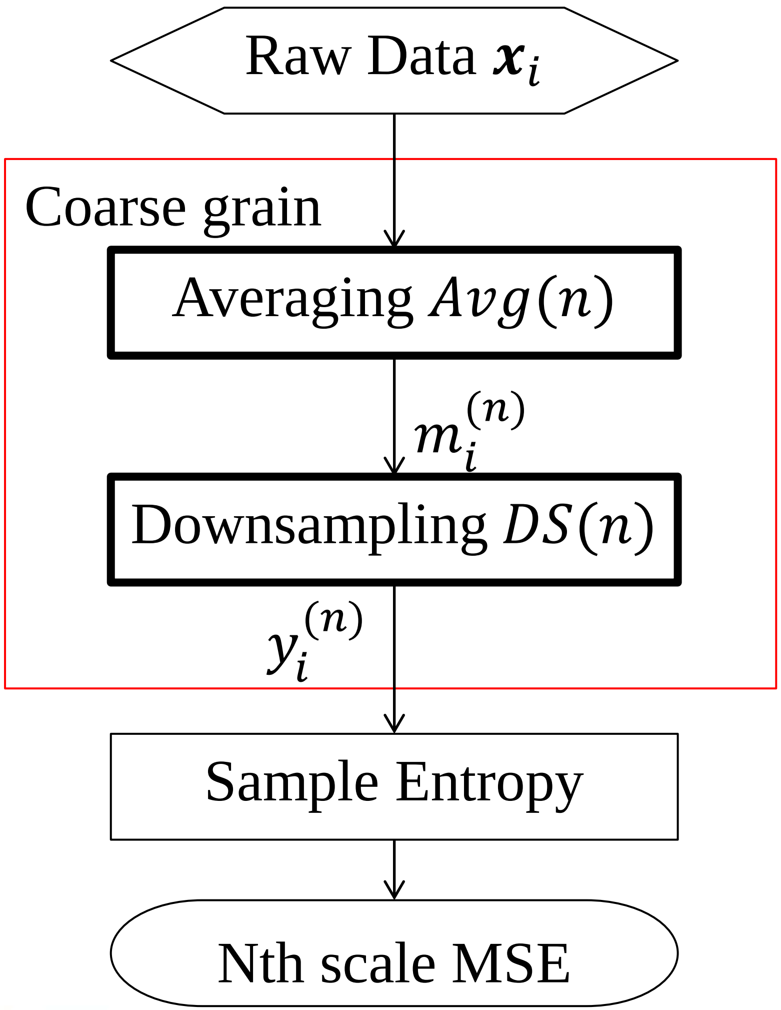
- Why different distant function?
  - To reduce the influence of noise.

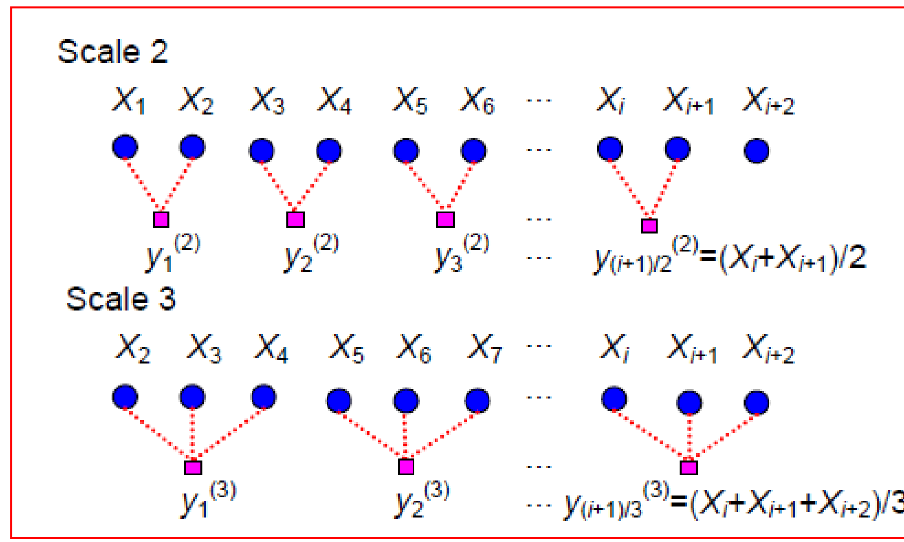
- Notes:
  - Typically, in order to get a reasonable sample entropy, data length of the target signal should be larger than 750.

Multi Scale Entropy (MSE)

- Reasons for using a multi-scale approach
  - It is essential to avoid artifacts of oversampling and noise.
  - different structures may emerge from a time series when observed at different time scales.
- Methodologies
  - Torres and Gamero (2000) using sliding windows of varying width to estimate a sort of "instantaneous complexity“
  - Costa et.al. (2002) downsample the time-series using a rectangular window which they move along the series in jumps equal to the window width.

Multi Scale Entropy (Algorithm)





$$y_i^{(\tau)} = \frac{1}{\tau}\sum_{n=(i-1)\tau+1}^{i\tau} x_n$$

Multi Scale Entropy
Coarse grain

- Average
  - Mean

$$m_i^{(n)} = \frac{1}{n}\sum_{k=i}^{i+n-1} x_k \qquad (1)$$

  - Binomial weighted

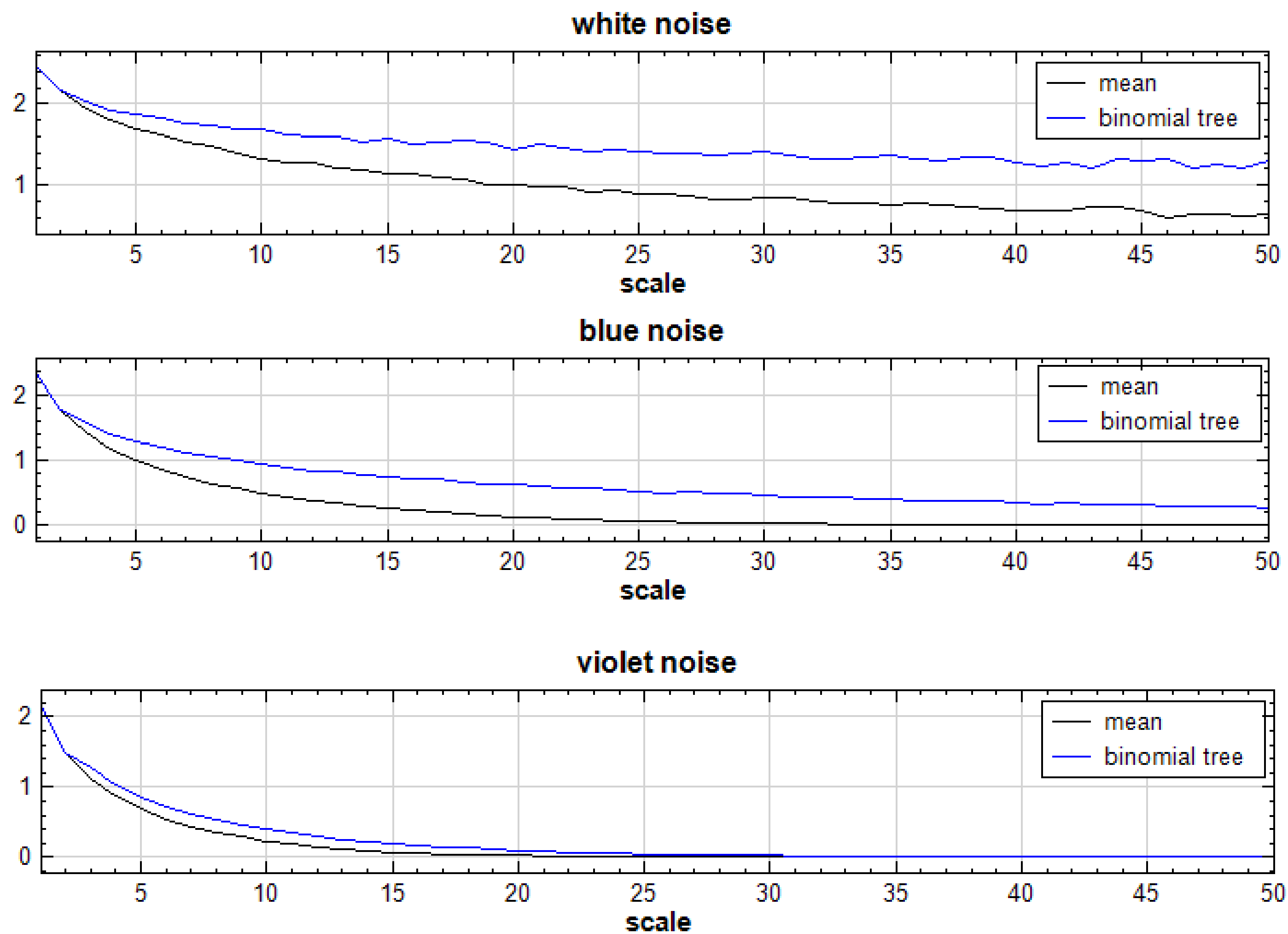
$$m_i^{(n)} = \frac{1}{2^{n-1}}\sum_{k=i}^{i+n-1} \binom{n}{k} x_k \quad \text{(Wu, 2011)} \qquad (2)$$

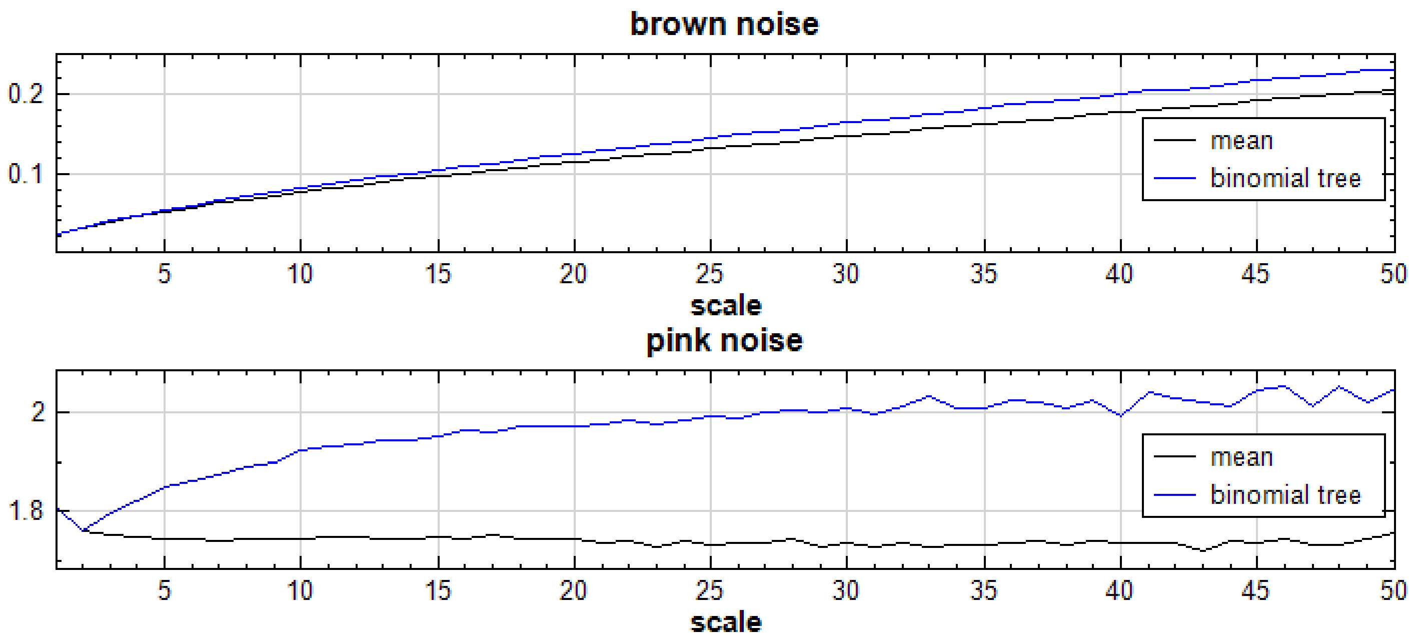
  - Gaussian window

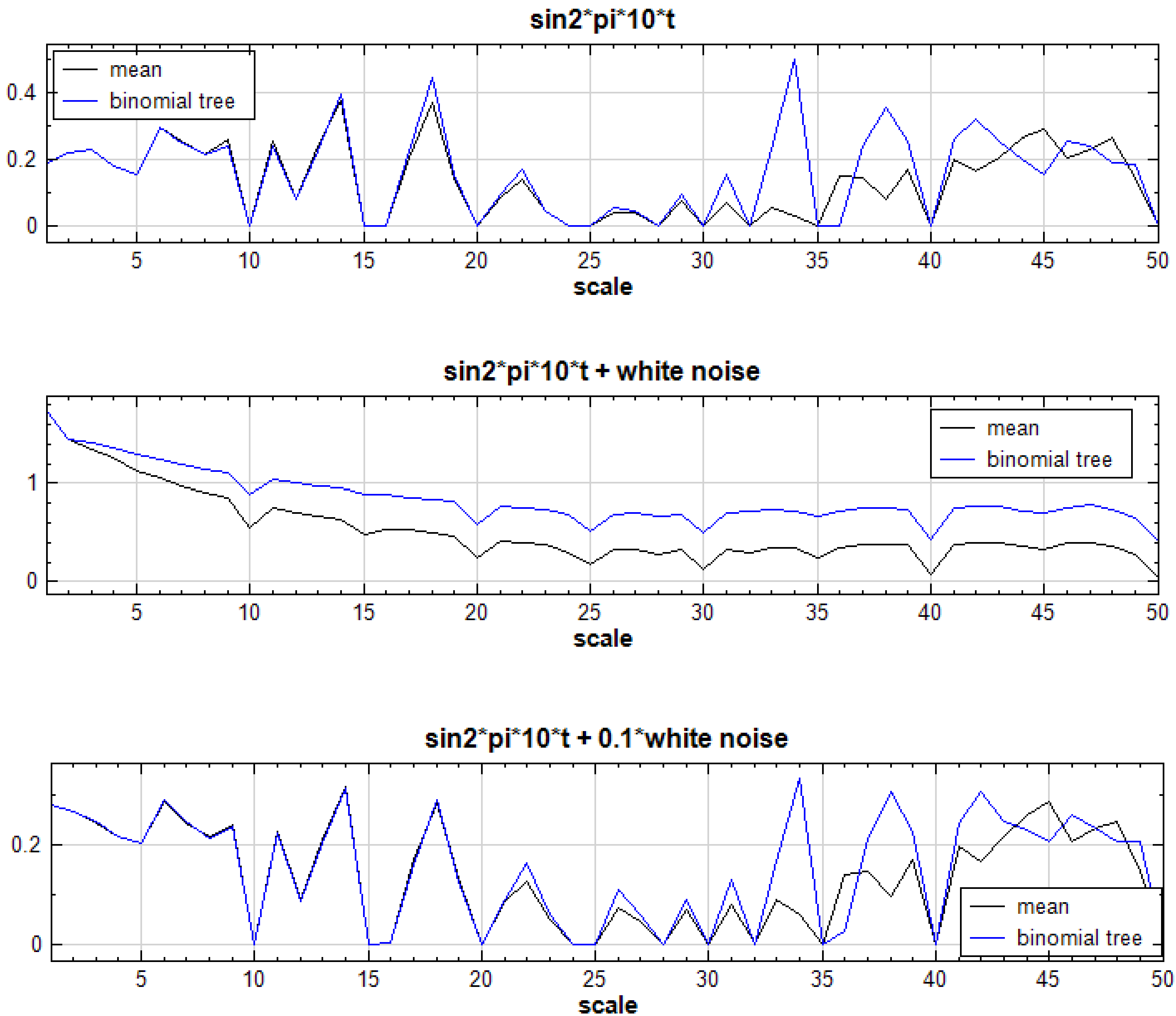
- Downsampling

$$y_i^{(n)} = m_{ni}^{(n)} \qquad (3)$$

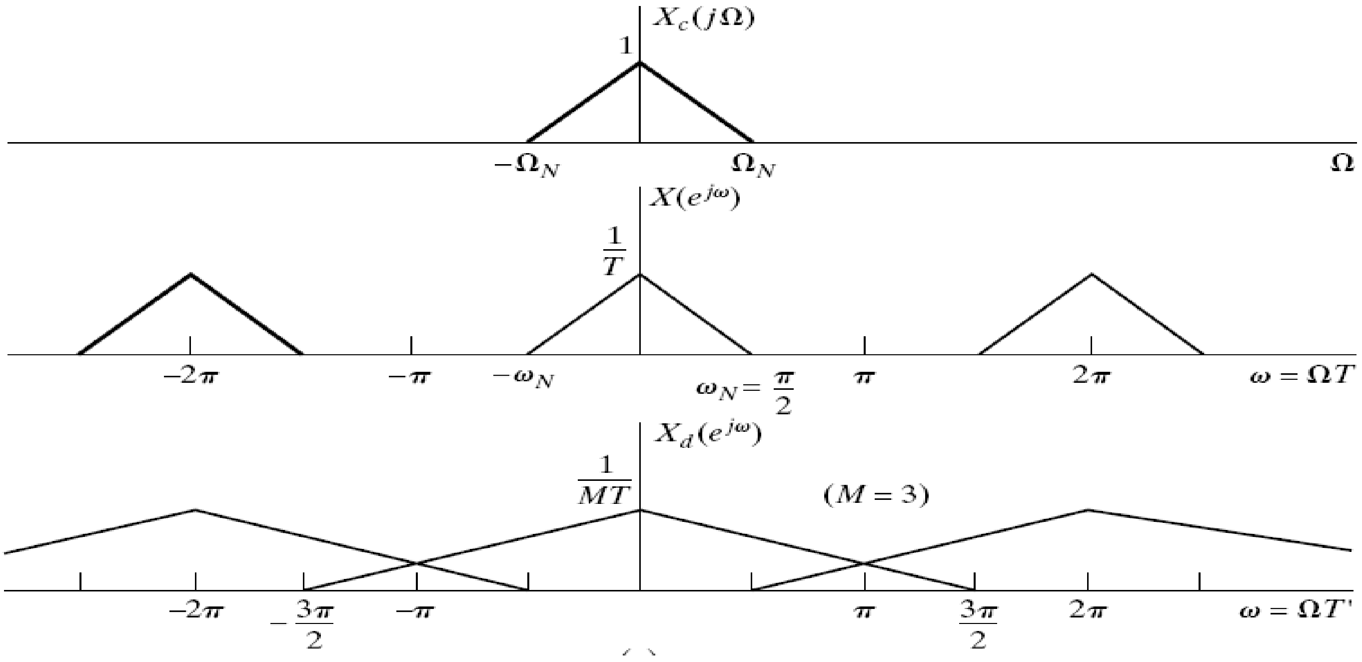
Multi Scale Entropy
Different Coarse grain method



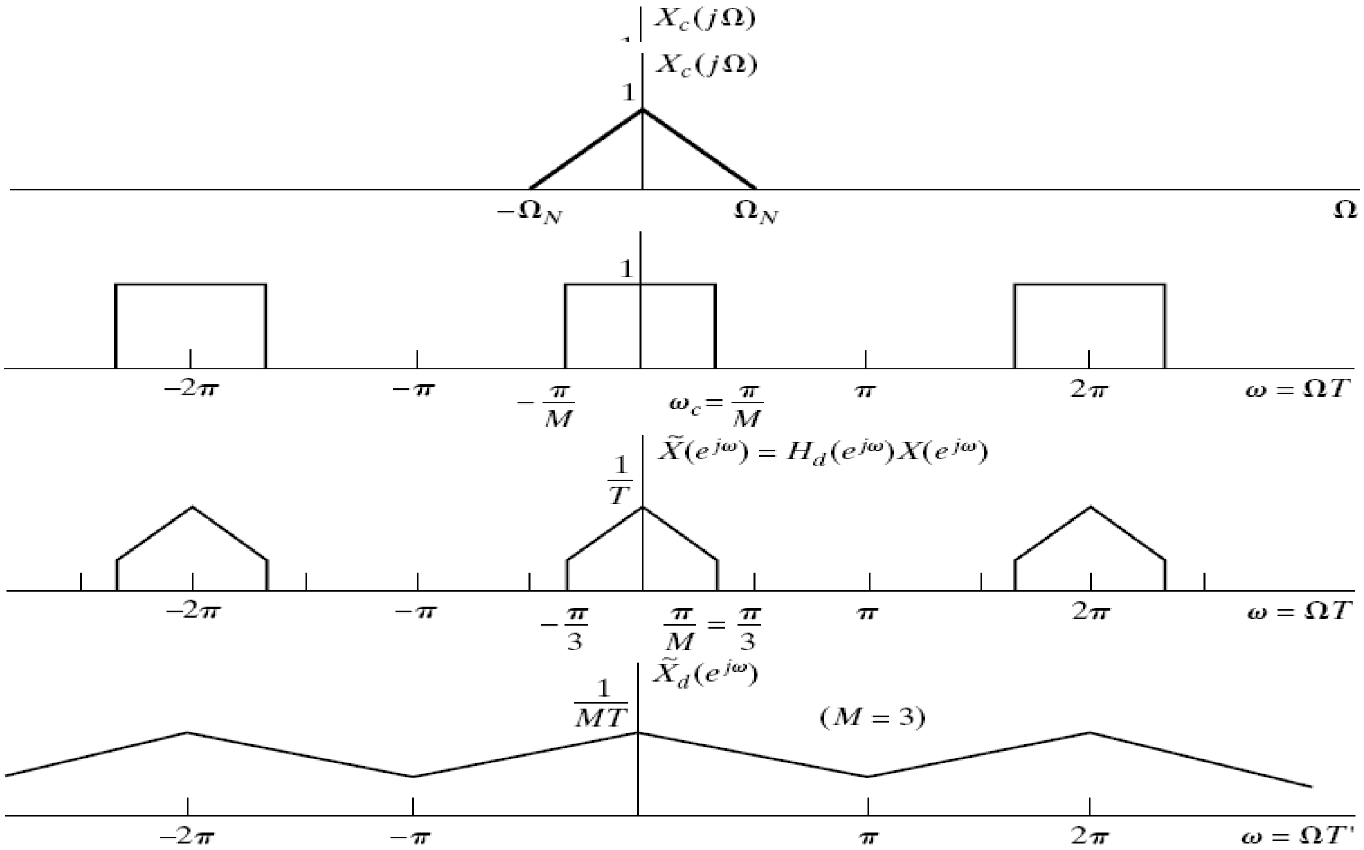

brown noise
0.2
0.1
5
10
15
20
25
30
35
40
45
50
scale
mean
binomial tree
pink noise
2
1.8
5
10
15
20
25
30
35
40
45
50
scale
mean
binomial tree


sin2*pi*10*t
mean
binomial tree
0.4
0.2
0
5
10
15
20
25
30
35
40
45
50
scale
sin2*pi*10*t + white noise
mean
binomial tree
1
0
5
10
15
20
25
30
35
40
45
50
scale
sin2*pi*10*t + 0.1*white noise
0.2
0
mean
binomial tree
5
10
15
20
25
30
35
40
45
50
scale

Down Sampling & Aliasing



Down Sampling & Aliasing

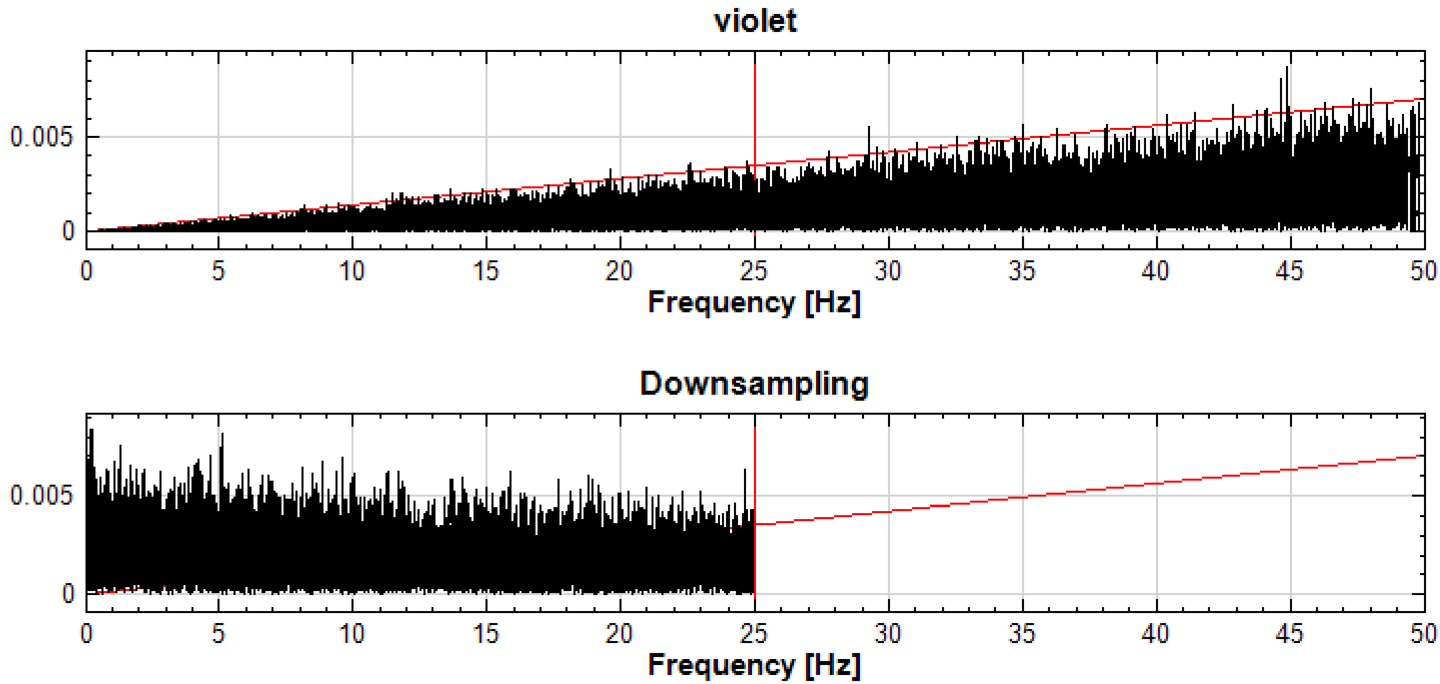


Downsampling - Aliasing

- Data ：$violet\ noise$ $\quad 0 < t < 1000$

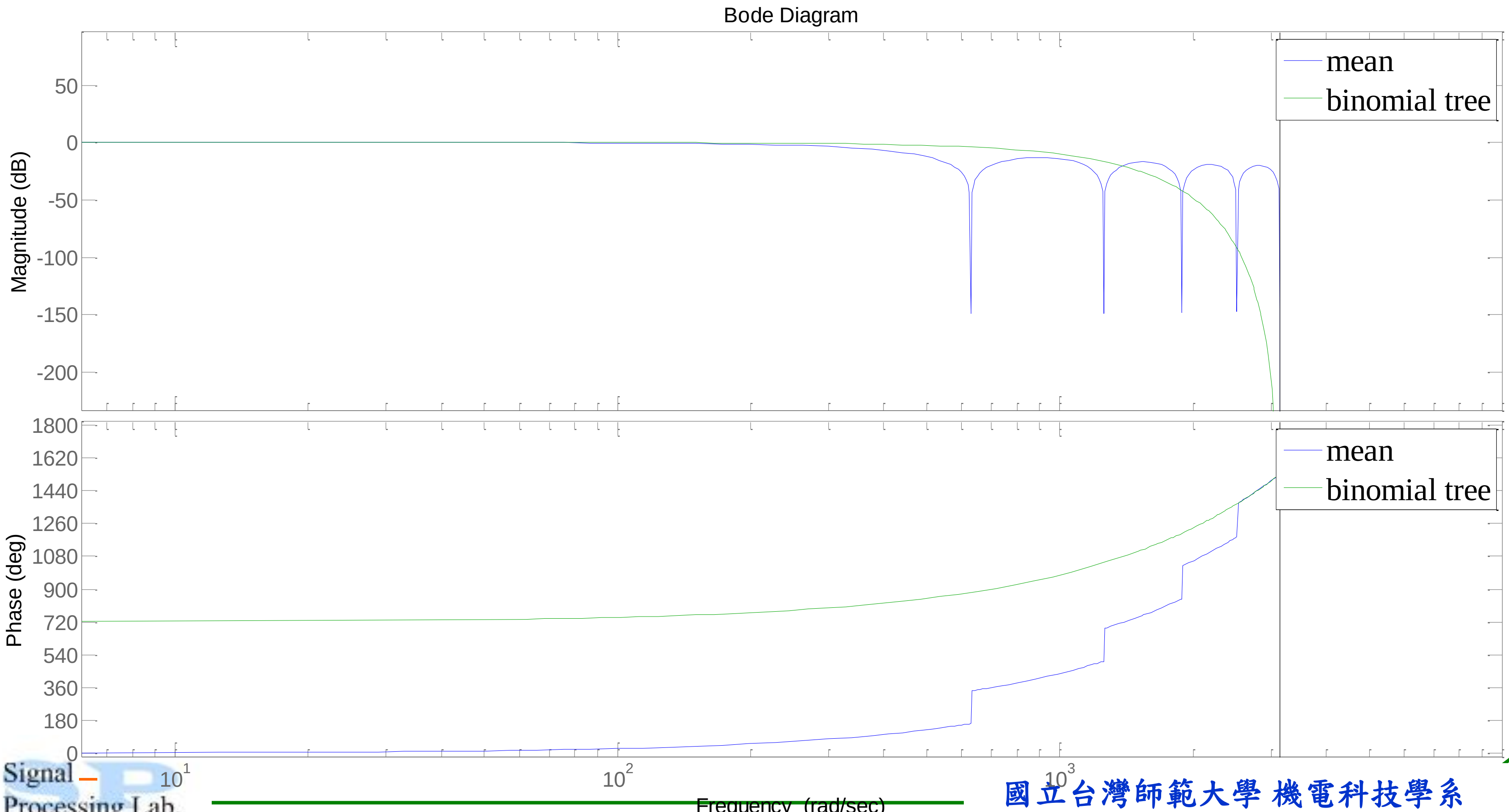
  $F[x_{violet}(t)]^2 \propto f^2$

- Sampling frequency：$100Hz$



Down Sampling & Aliasing

- 10th level averaging (algebraic mean v.s. binomial tree)



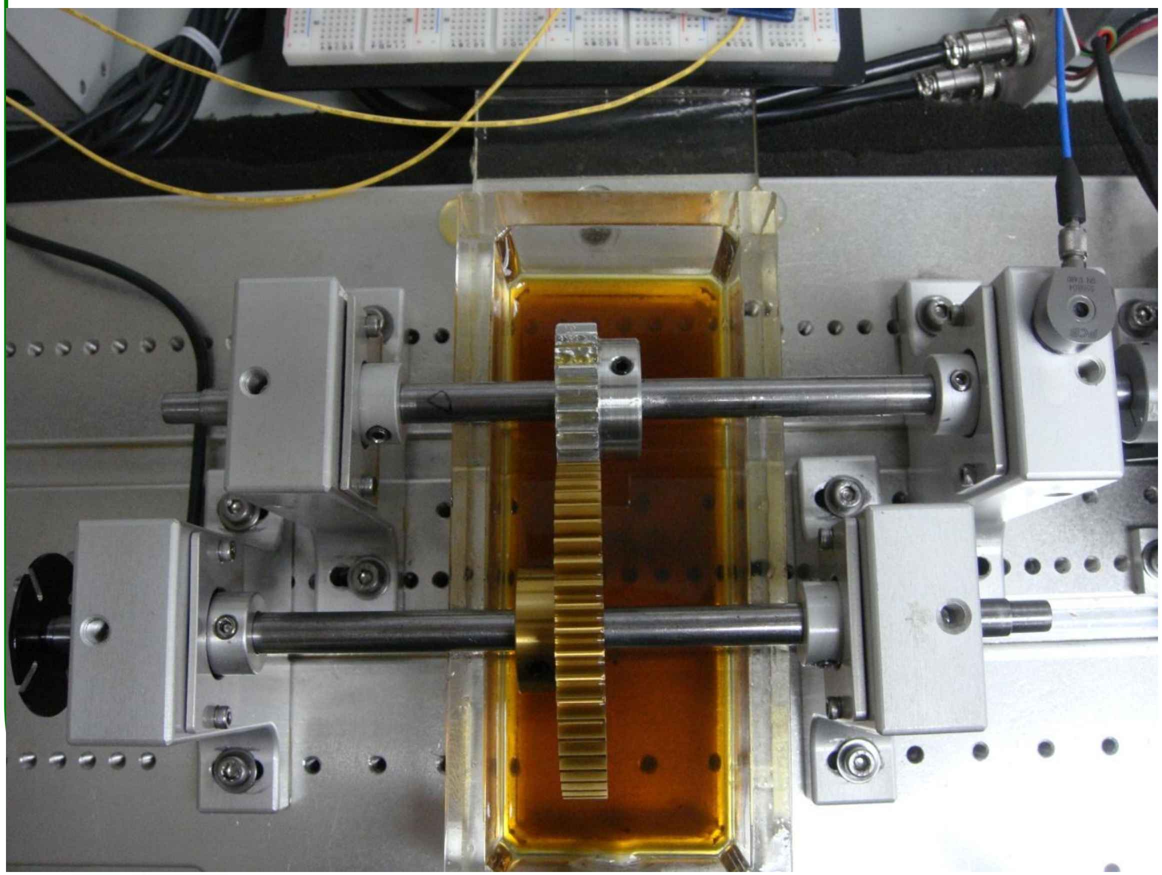
Applications of MSE

- Bio-signal
  - EEG
  - Heart Rate Dynamics
  - human gait dynamics.
- Vibrational signal of machine
  - Motor shaft misaligment detection
  - Bearing Fault diagnosis
  - composite milling

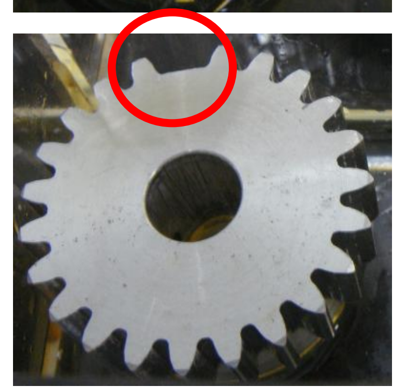
Complexity Measurement for vibration signal of machines

- Lempel-Ziv Complexity
  - Yan & Gao 2004, machine healthy evaluation
  - Hong & Liang 2008, fault severity assessment
- Multi scale entropy
  - Lin et.al 2010 , Motor shaft misaligment detection
  - Zhang et.al 2010, Bearing Fault diagnosis
  - Litak et.al 2011, composite milling

MSE for vibration signals
ITRI's Data Set



正常



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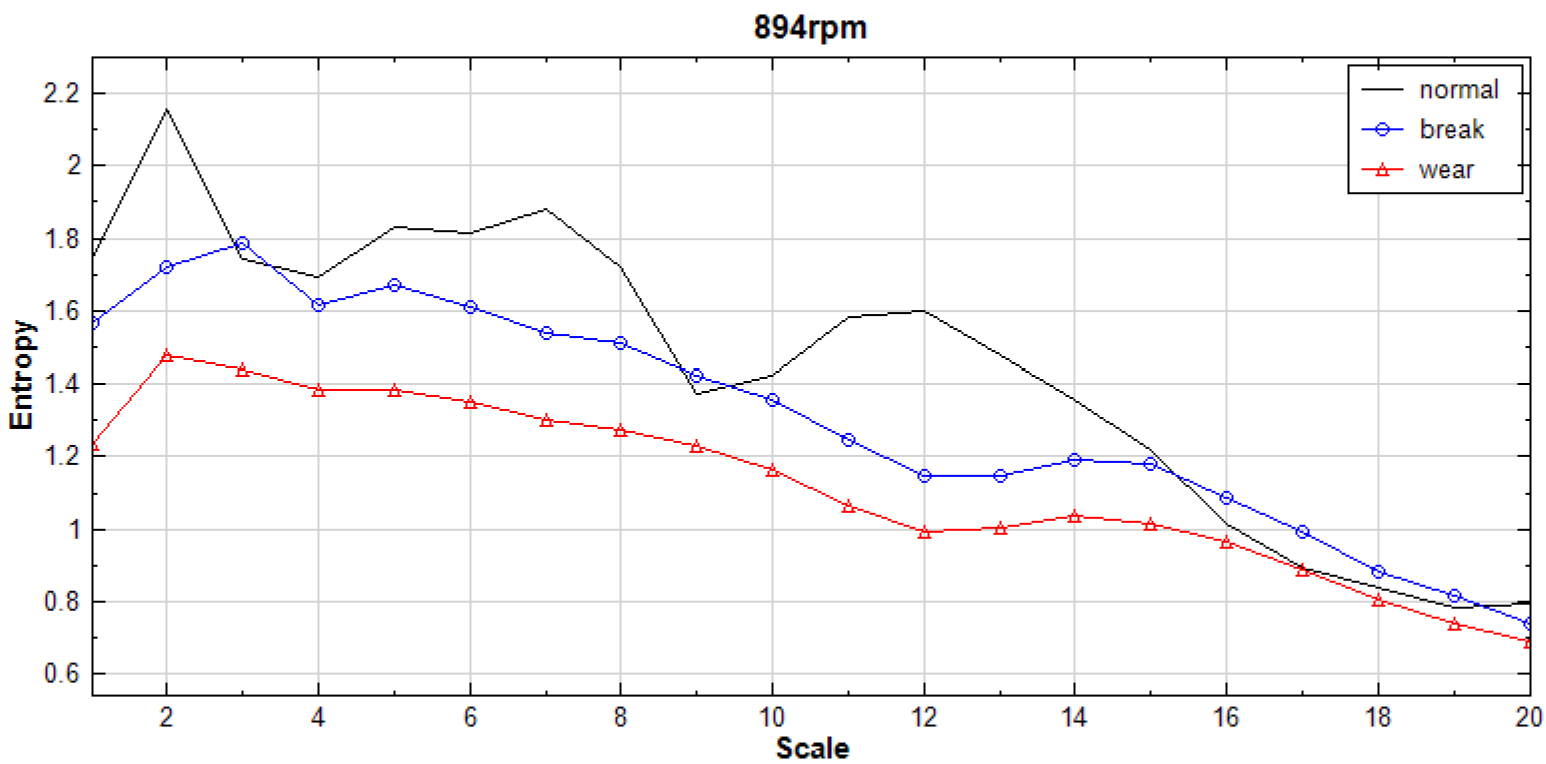
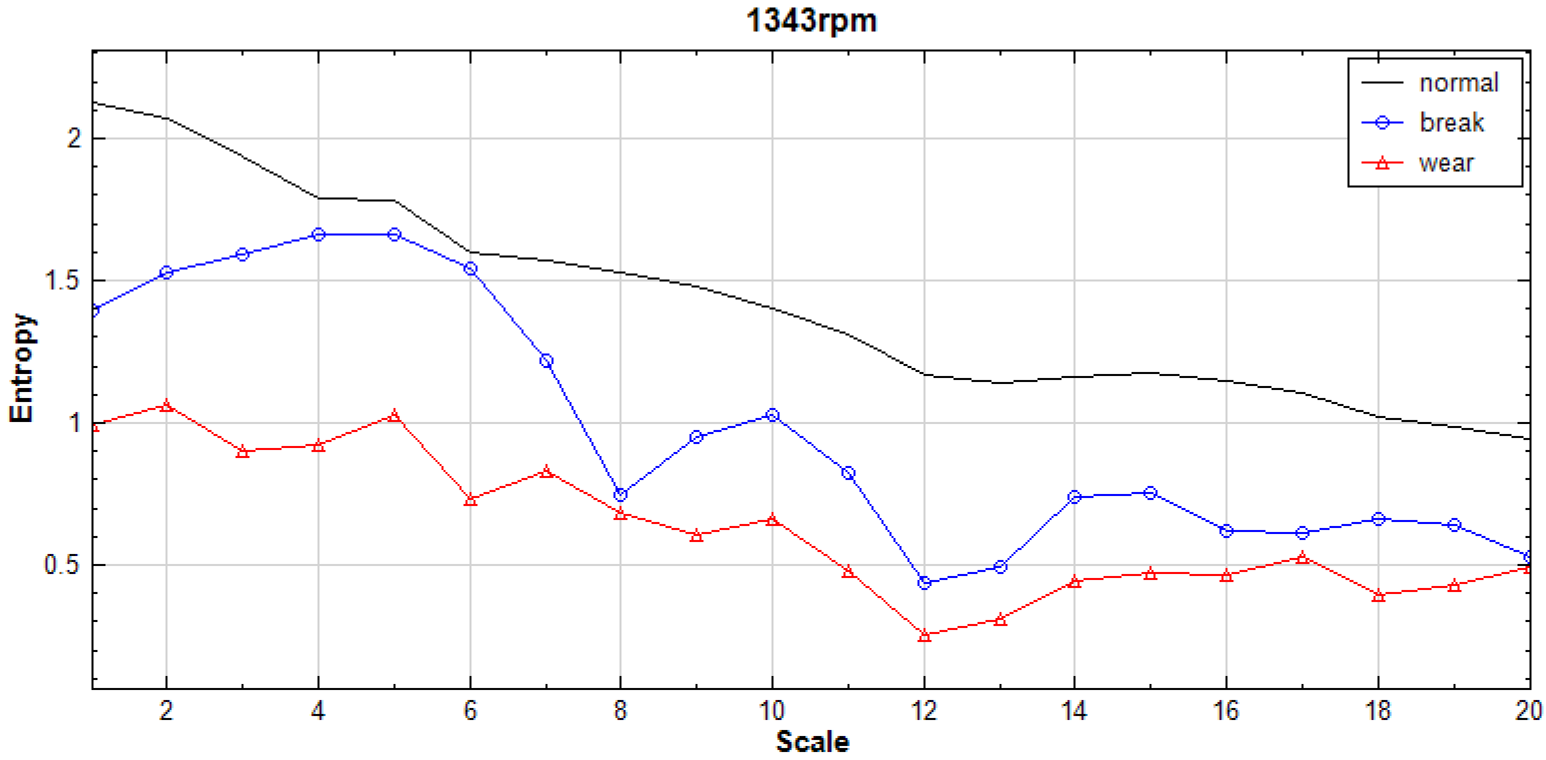
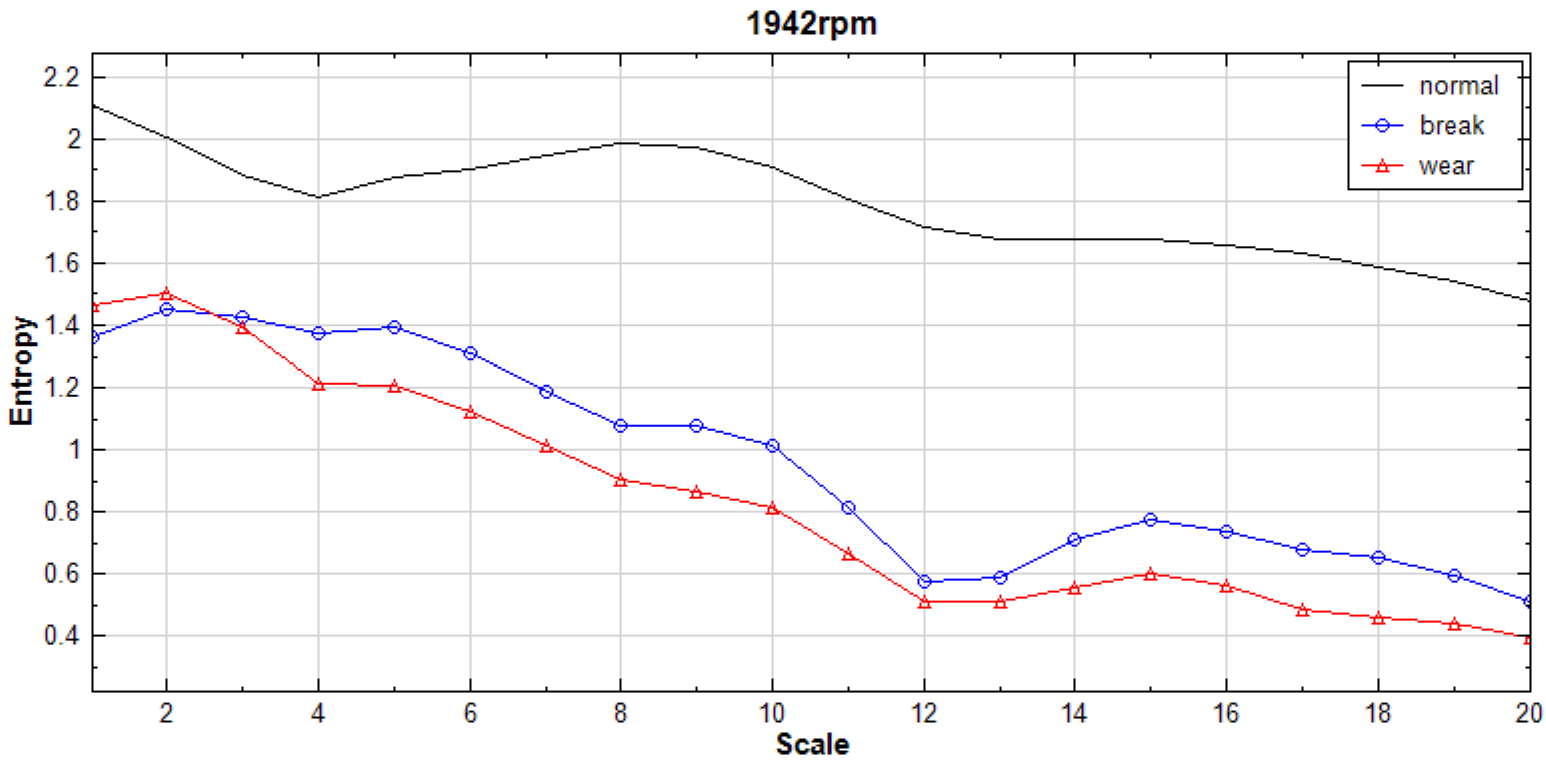
MSE for vibration signals ITRI's Data Set



MSE for vibration signals ITRI's Data Set



MSE for vibration signals ITRI's Data Set

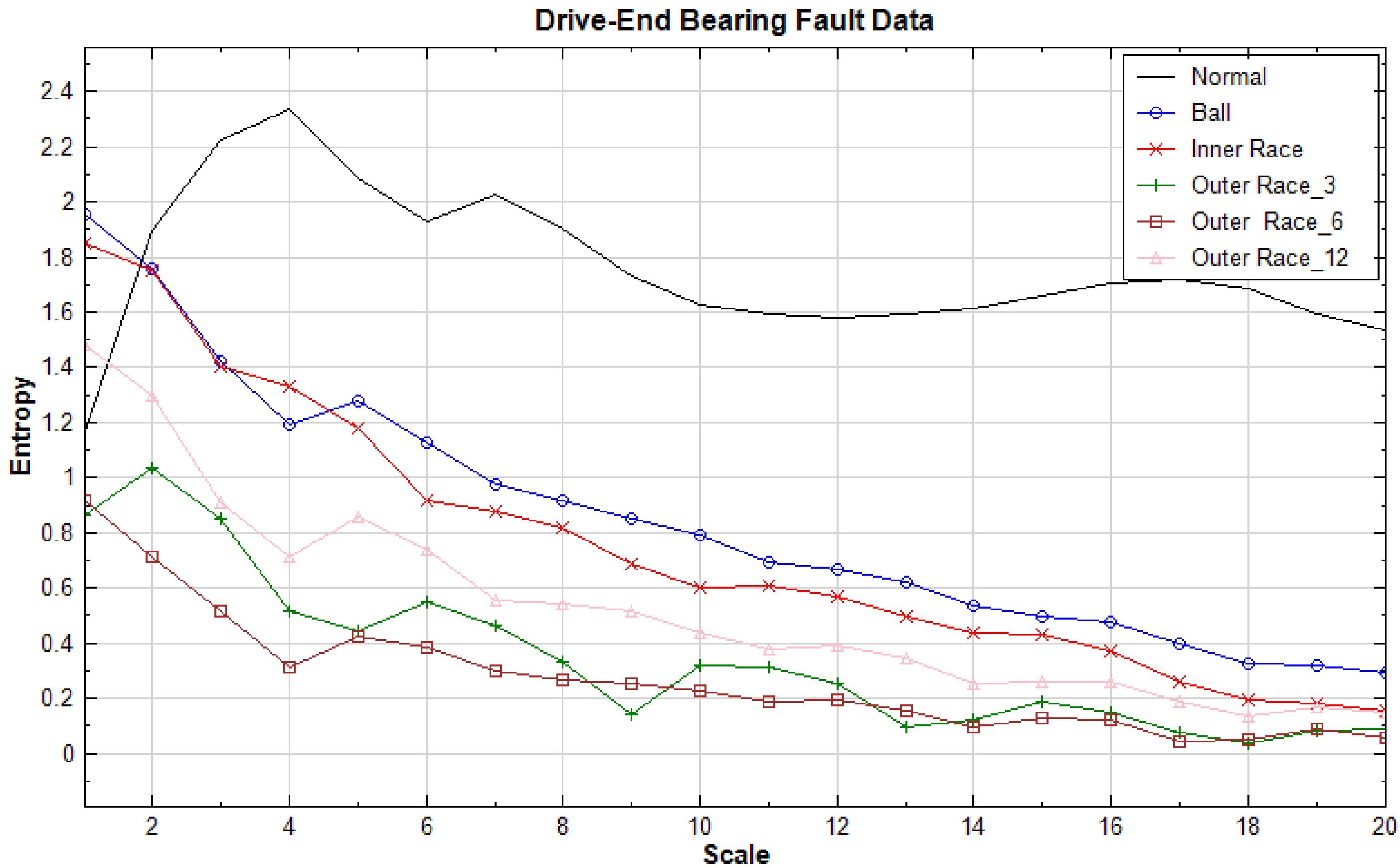


MSE for vibration signals CWRU's Data Set

- Data Source
  - Case Western Reserve University Bearing Data Center
- 6 different classes
  - Normal
  - Ball
  - Inner Race
  - Outer Race_3(錯誤在3點鐘方向)
  - Outer Race_6(錯誤在6點鐘方向)
  - Outer Race_12(錯誤在12點鐘方向)

MSE for vibration signals CWRU's Data Set

- MSE for different classes (data length=120000)

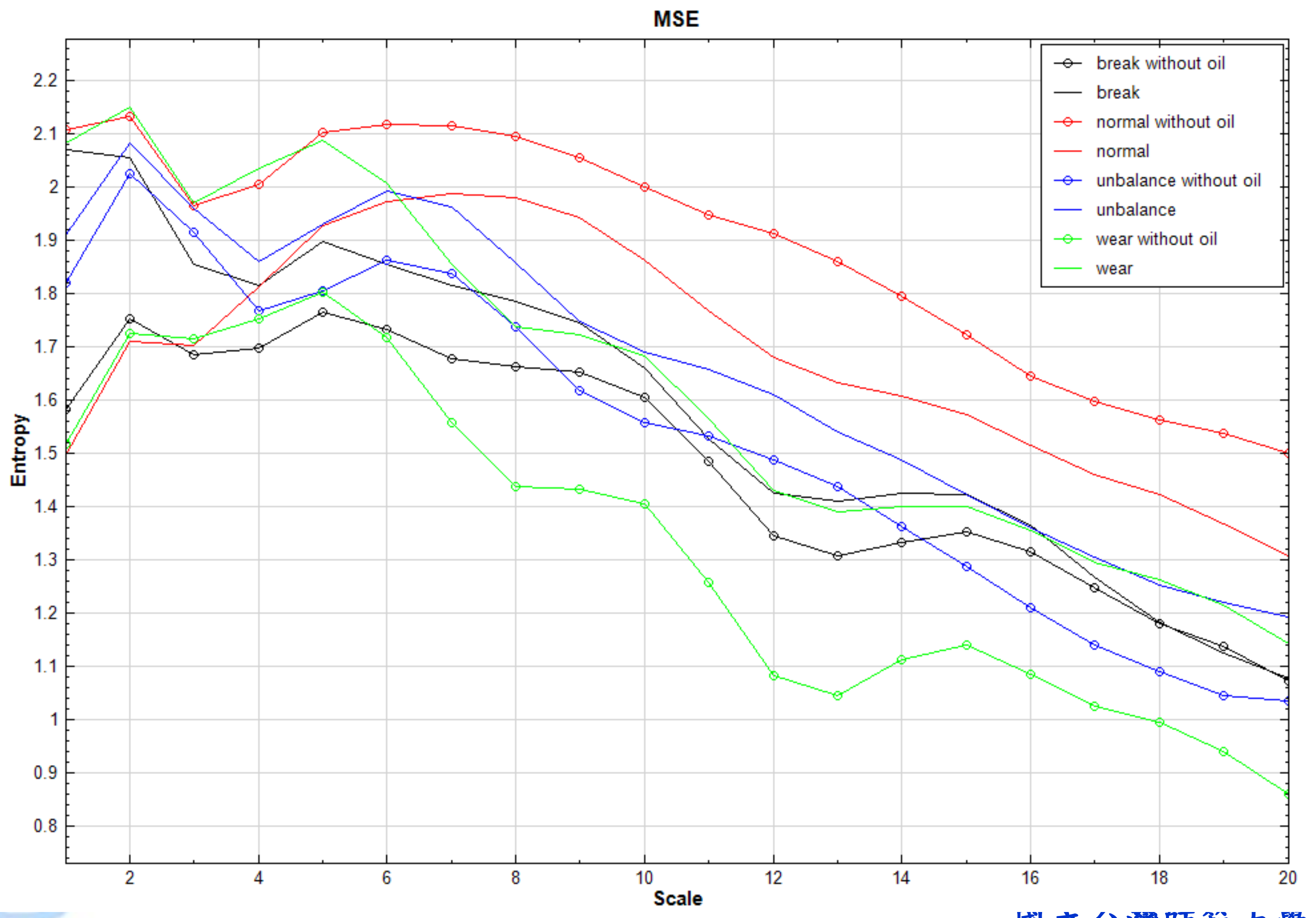


Fault Detection System based on MSE, FFT and SVM

- Data Source ITRI
- 8 different classes
  - Normal with oil
  - Normal without oil
  - Break with oil
  - Break without oil
  - Wear with oil
  - Wear without oil
  - Unbalanced with oil
  - Unbalanced without oil

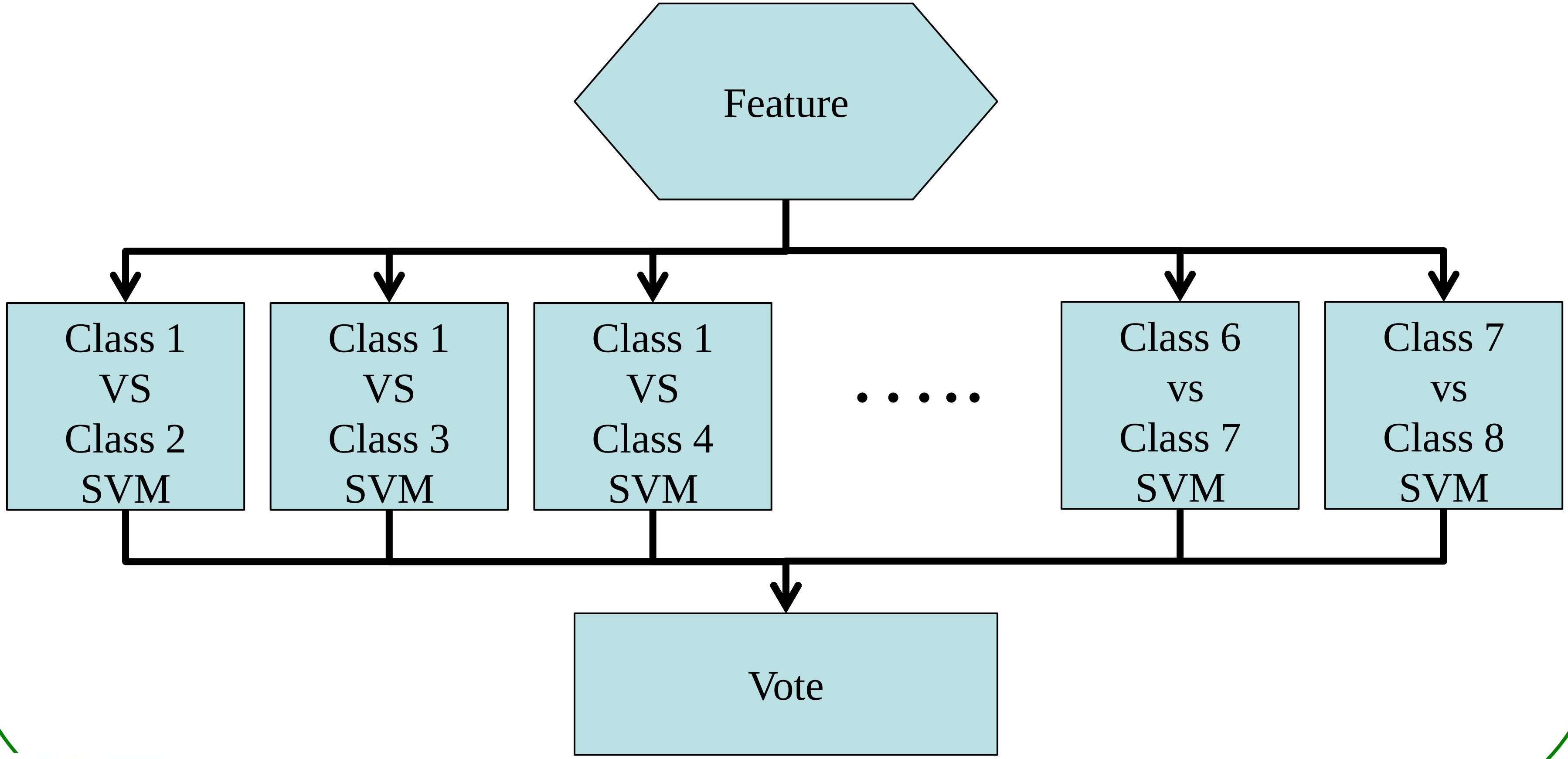
Fault Detection System based on MSE, FFT and SVM

- MSE for different classes (data length=128000)



Fault Detection System based on MSE, FFT and SVM

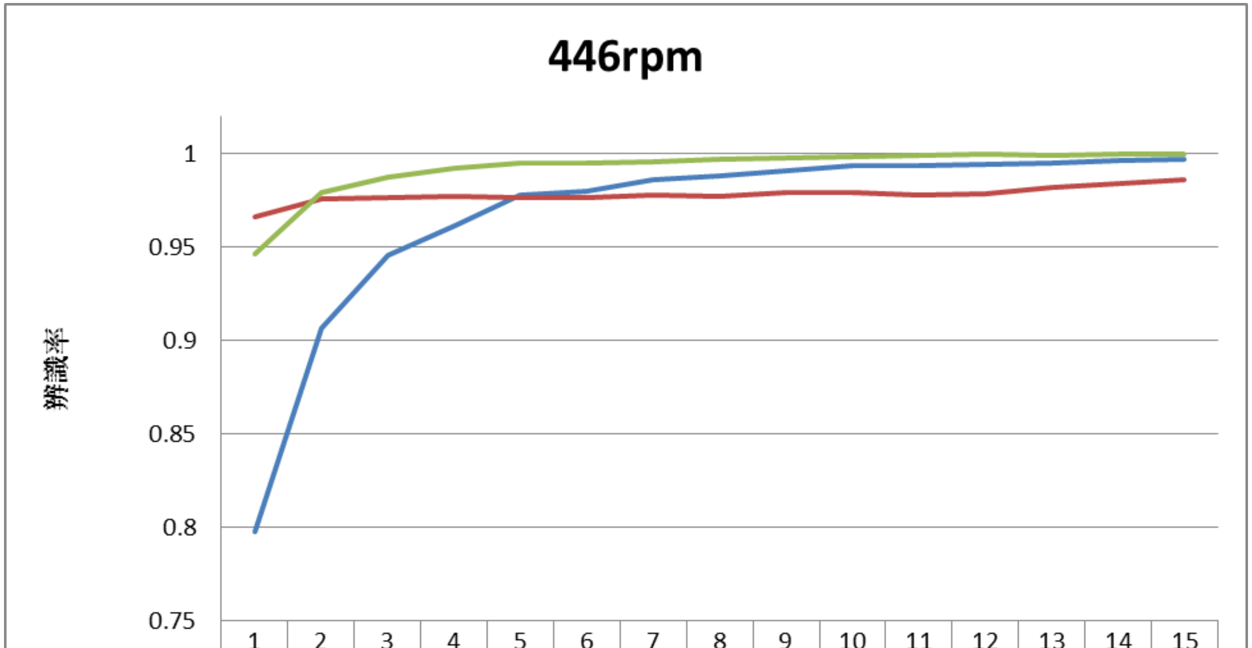
- Experiment flow



Fault Detection System based on MSE, FFT and SVM

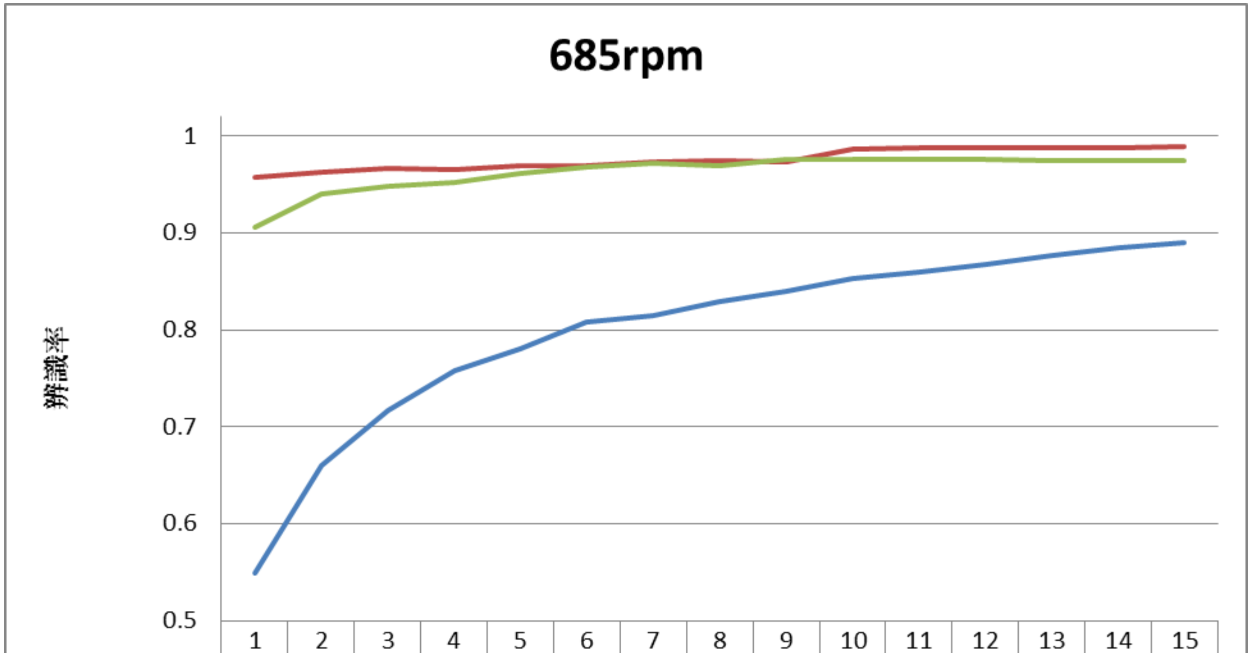
- Data length for each training data & testing data: 16384
- Number of training data for each class: 11
- Number of testing data for each class: 12
- Features:
  - MSE (15 scales)
  - FFT (8192 bins)
- **Feature Selection**
- Recognition rate for each experimental case is the average of 200 trials.

Fault Detection System based on MSE, FFT and SVM



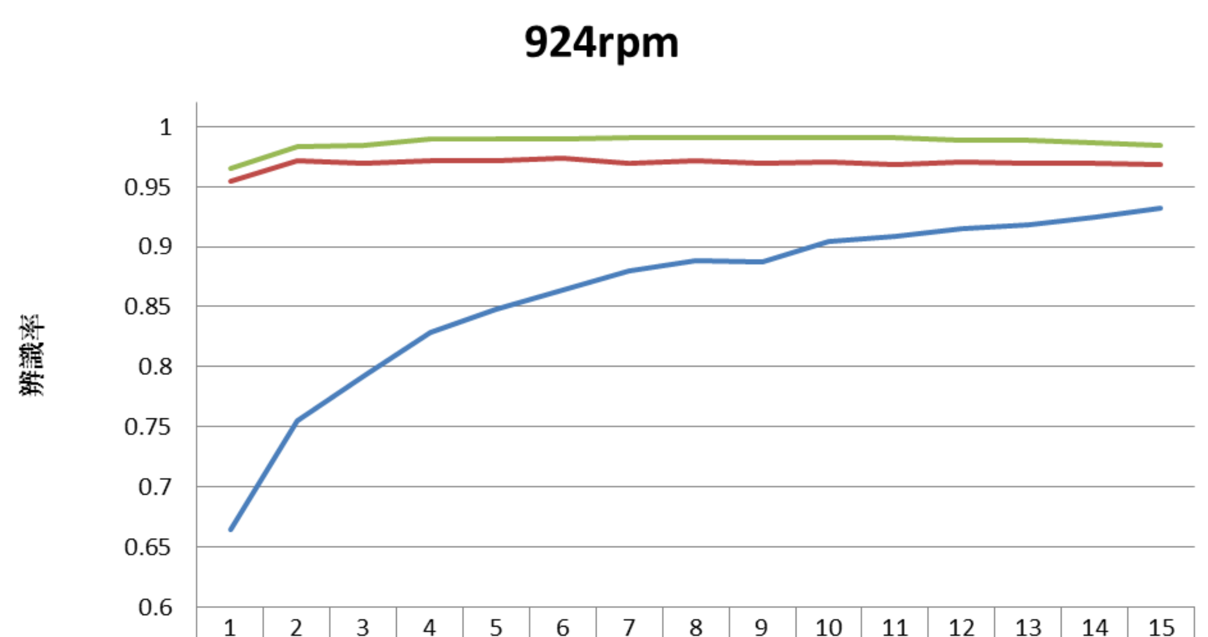
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
FFT	0.797	0.907	0.946	0.961	0.978	0.98	0.986	0.988	0.991	0.993	0.994	0.994	0.995	0.996	0.997
MSE	0.966	0.976	0.977	0.977	0.976	0.977	0.978	0.977	0.979	0.979	0.978	0.978	0.982	0.984	0.986
MSE+FFT	0.947	0.979	0.987	0.992	0.995	0.995	0.996	0.997	0.998	0.998	0.999	1	0.999	1	1

Fault Detection System based on MSE, FFT and SVM



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
FFT	0.549	0.66	0.717	0.758	0.781	0.808	0.815	0.83	0.84	0.853	0.859	0.868	0.876	0.884	0.89
MSE	0.958	0.963	0.967	0.965	0.969	0.969	0.974	0.975	0.973	0.987	0.988	0.987	0.988	0.987	0.989
MSE+FFT	0.906	0.94	0.948	0.953	0.961	0.968	0.971	0.969	0.975	0.975	0.975	0.975	0.975	0.974	0.974

Fault Detection System based on MSE, FFT and SVM



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
FFT	0.665	0.755	0.792	0.829	0.848	0.864	0.88	0.888	0.887	0.904	0.908	0.915	0.918	0.925	0.932
MSE	0.955	0.971	0.97	0.971	0.971	0.974	0.969	0.972	0.969	0.971	0.969	0.97	0.969	0.97	0.969
MSE+FFT	0.966	0.983	0.984	0.99	0.99	0.99	0.991	0.991	0.991	0.991	0.991	0.989	0.989	0.986	0.984

Fault Detection System based on MSE, FFT and SVM

- MSE在計算規律訊號與敲擊訊號時，明顯有不同的結果，很適合作為分辨兩類的特徵
- 由辨識結果可以發現，**MSE只需要少量的特徵，便可以得到很高的辨識率**
- 比較MSE的辨識結果與FFT的辨識結果可以發現，在**中轉速與高轉速的時候MSE的結果明顯優於FFT的結果**，在低轉速時兩者的結果也相去不遠
- 若綜合MSE與FFT的特徵進行辨識，可以進一步提高辨識率

Future researches

- Apply different distance function to calculate the multi scale entropy.
  - To overcome the noise corrupted in vibration signal
- Apply different averaging function
- Different multi-scale approaches
  - Wavelet
  - EMD

敬請批評與指教

Thanks for your attention!