希爾伯特黃轉換簡介
(Hilbert Huang Transform)

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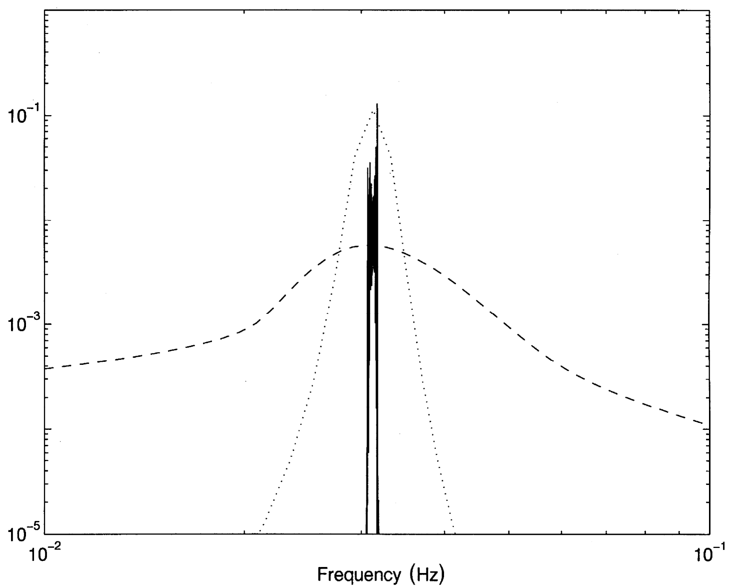
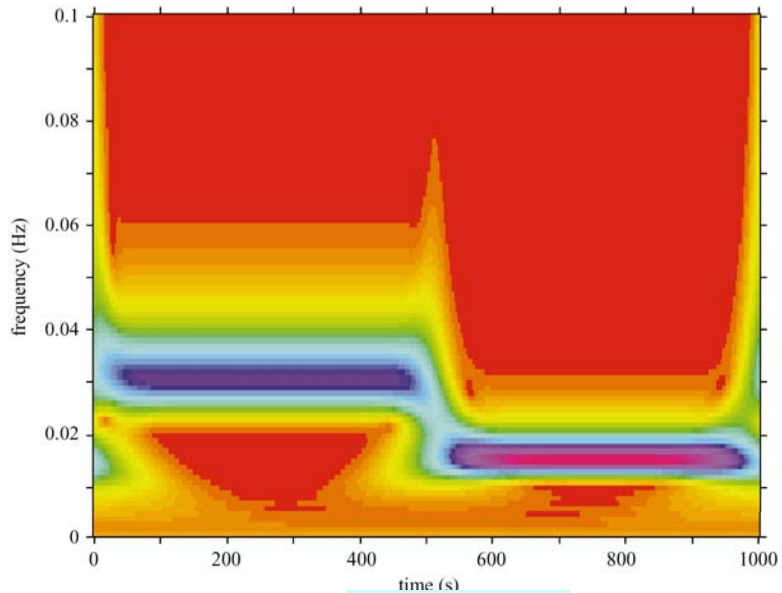
🕮 前言

- 在訊號處理與頻譜分析的目的是要描述信號的頻譜含量在時間上變化，以便能在時間和頻譜上同時表示信號的能量或者強度。

- 傅立業頻譜並沒有告訴我們哪些頻率在什麼時候出現。此一方法無法表現出也無法表現資料的時變性





time

📖 黃鍔博士（Norden E. Huang）簡介

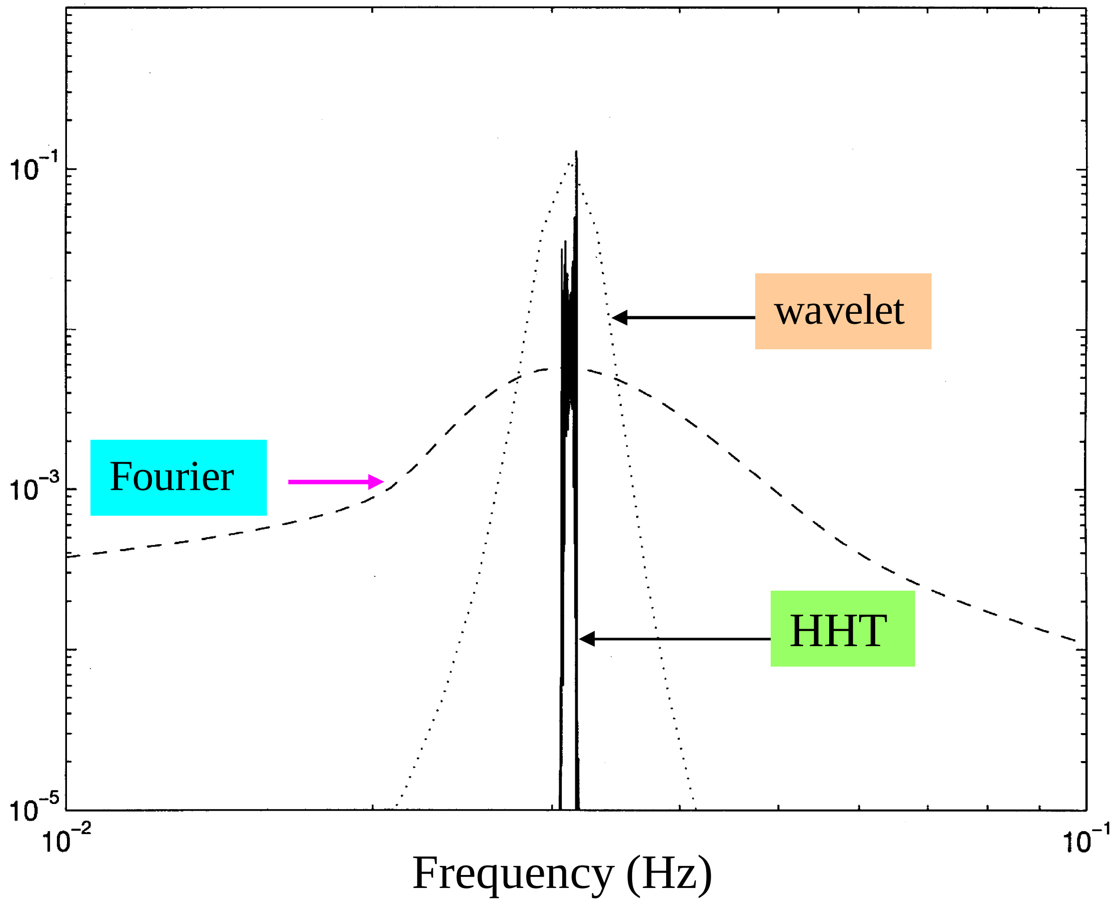
- 1937 年出生于湖北
- 新竹中學畢業
- 1960 年從臺灣大學土木系畢業
- 1962 年進入美國約翰・霍普金斯大學力學系
- 華盛頓大學海洋地理學系研究員
- 北卡羅來納州立大學海洋地理學系副教授
- 1975 年起進入美國太空總署(NASA)
- 加州理工學院 (CIT) 客座教授 ；哈佛醫學院客座研究員
- 美國國家工程學院院士
- 2003 年美國 NASA 發明獎
- 2004 中央研究院院士 (第二十五屆)

🕮 希爾伯特-黃轉換 (Hilbert Huang Transform) 理論簡介

- Hilbert-Huang (HHT) 轉換方法是黃鍔根據近代知名數學家 Hilbert 的數學理論設計，做爲分析非穩定或非線性的訊號

Comparisons among the Fourier, marginal Hilbert and wavelet spectra

spectra

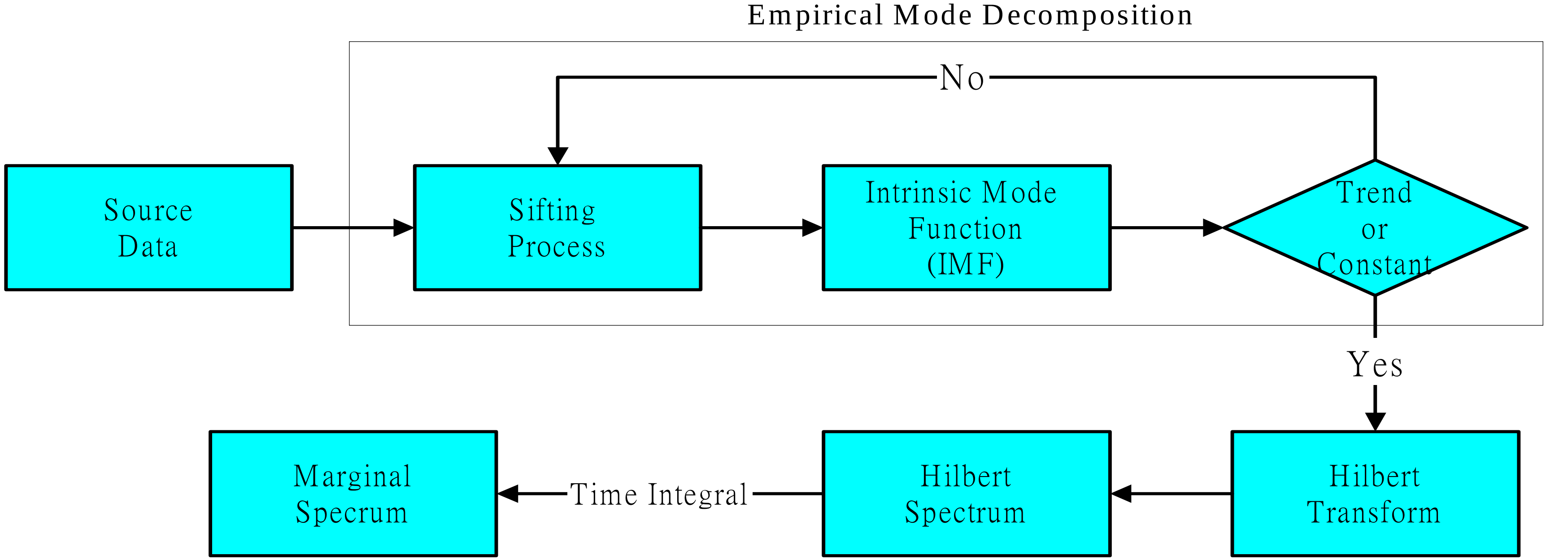


應用範圍

- 哈佛醫學院用 HHT 來測量心律不整
- 約翰霍普金斯公共衛生學院用它來測量登革熱的擴散
- 美國聯邦調查局用 HHT 來辨識發言者的身分
- 海軍用它來探測潛艇
- 聯邦公路管理局研究中心測量公路、橋梁的安全
- 地震工程、地球物理探測、衛星資料分析
- 潛艇設計、結構損害偵測
- 血壓變化和心律不整
- 潮汐、波浪場等各項研究

希爾伯特-黃轉換處理架構流程圖

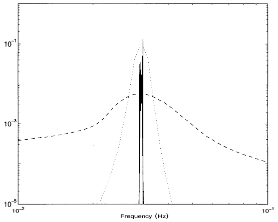
Empirical Mode Decomposition, EMD

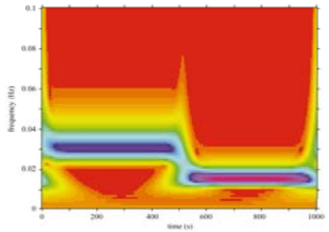


$$h(\omega) = \int_0^T H(\omega, t)dt$$

$$X(t) = \sum_{j=1}^{n} a_j(t) e^{i2\pi \int f_j(t)dt}$$

$$Y(t) = \frac{1}{\pi} PV \int_{-\infty}^{\infty} \frac{X(t')}{t - t'} dt'$$

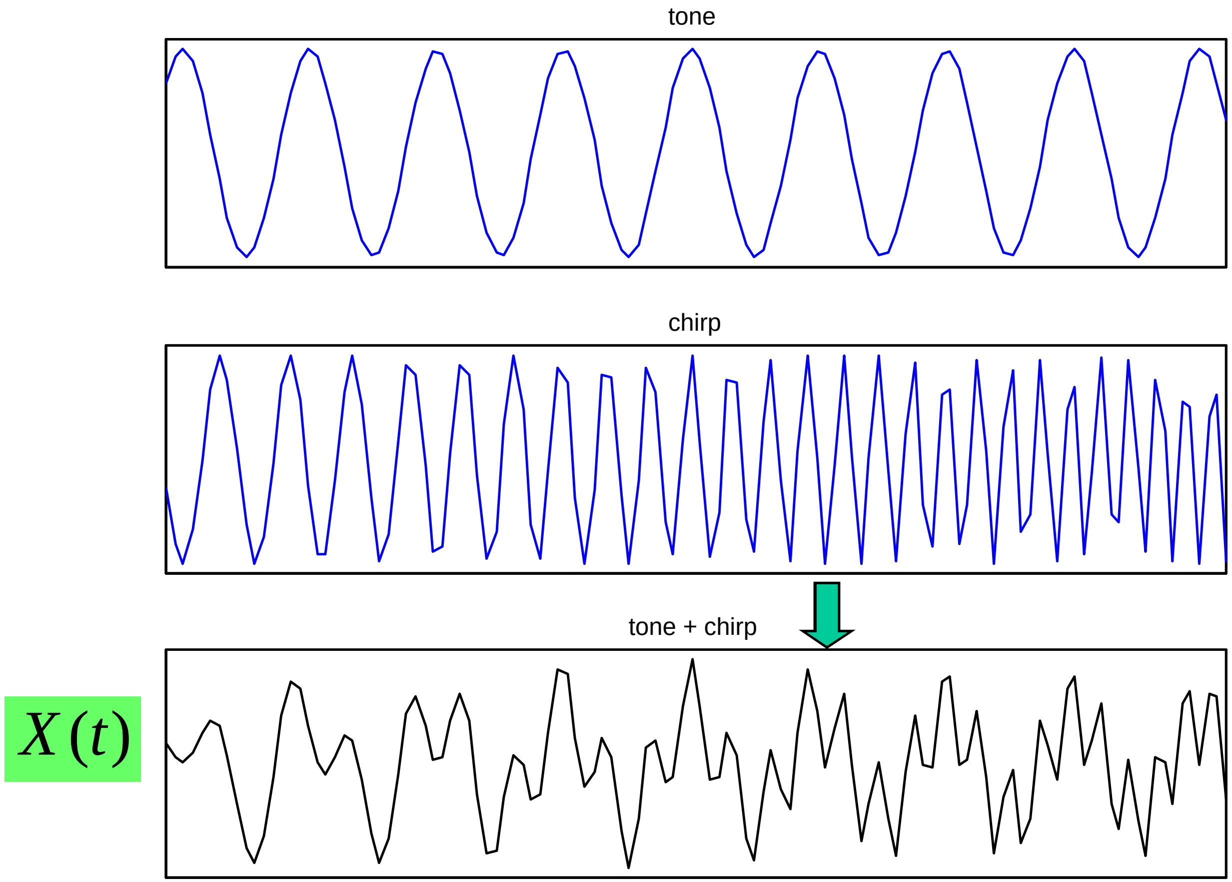




📖 EMD 過程(Empirical Mode Decomposition, EMD)簡介

	IMF1	IMF2	IMF3		IMF n
	$X(t)$	$X(t)-c_1=r_1$	$r_1-c_2=r_2$		$r_{n-2}-c_{n-1}=r_{n-1}$
0	$X(t)-m_1=h_1$	$r_1-m_2=h_2$	$r_2-m_3=h_3$		$r_{n-1}-m_n=h_n$
1	$h_1-m_{11}=h_{11}$	$h_2-m_{21}=h_{21}$	$h_3-m_{31}=h_{31}$		$h_n-m_{n1}=h_{n1}$
2	$h_{11}-m_{12}=h_{12}$	$h_{21}-m_{22}=h_{22}$	$h_{31}-m_{32}=h_{32}$		$h_{n1}-m_{n2}=h_{n2}$
3	$h_{12}-m_{13}=h_{13}$	$h_{22}-m_{23}=h_{23}$	$h_{32}-m_{33}=h_{33}$		$h_{n2}-m_{n3}=h_{n3}$
..	....				
k	$h_{1(k-1)}-m_{1k}=h_{1k}$	$h_{2(k-1)}-m_{2k}=h_{2k}$	$h_{3(k-1)}-m_{3k}=h_{3k}$		$h_{n(k-1)}-m_{nk}=h_{nk}$
IMF	$h_{1k}=c_1$	$h_{2k}=c_2$	$h_{3k}=c_3$		$h_{nk}=c_n$

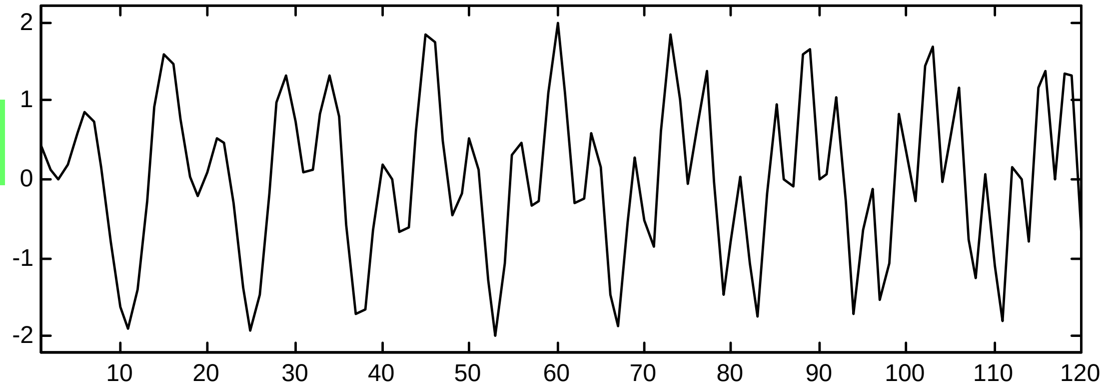
原始訊號



📖 EMD 過程

IMF1, iteration 0

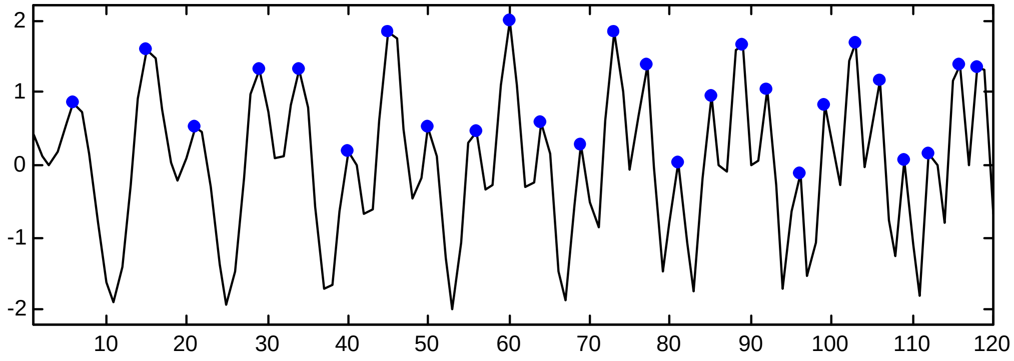
$X(t)$



EMD 過程

IMF1, iteration 0

$X(t)$

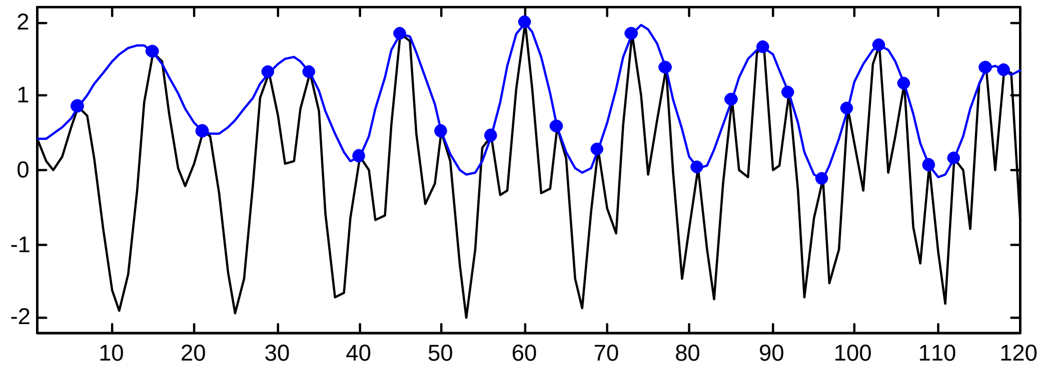


Step1：找出局部極大值

📖 EMD 過程

IMF1, iteration 0

$X(t)$

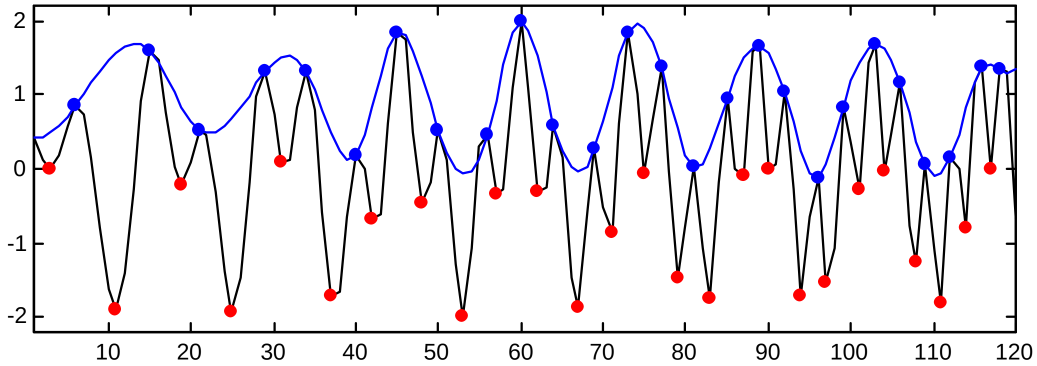


Step2：找出局部極大值的包絡線

📖 EMD 過程

IMF1, iteration 0

$X(t)$

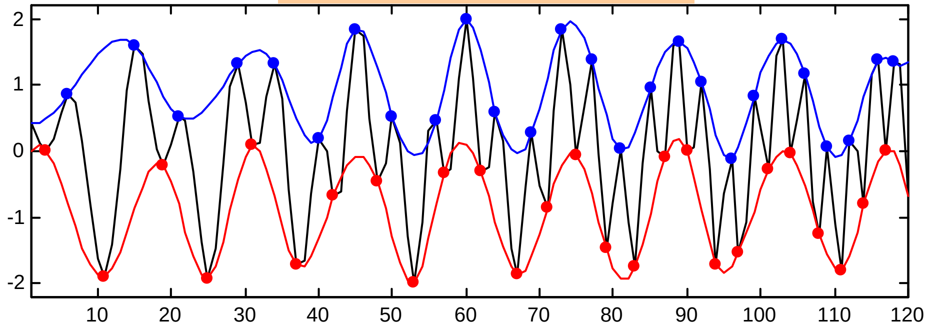


Step3：找出局部極小值

📖 EMD 過程

IMF1, iteration 0

$X(t)$

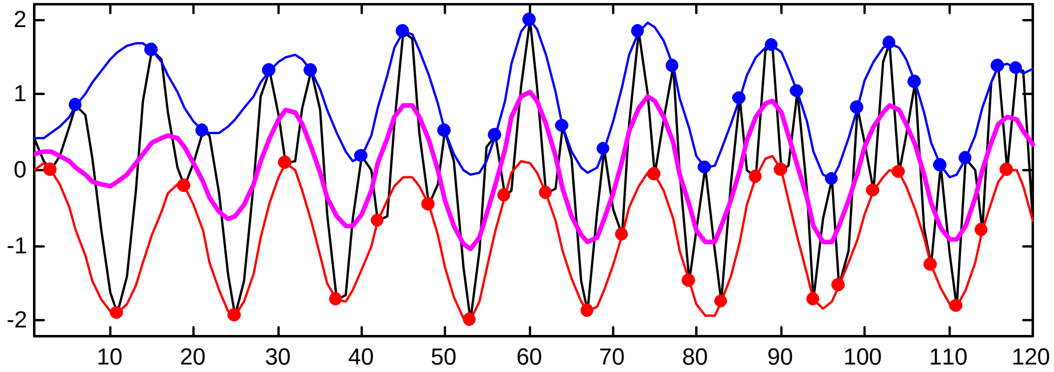


Step4：找出局部極小值的包絡線

📖 EMD 過程

IMF1, iteration 0

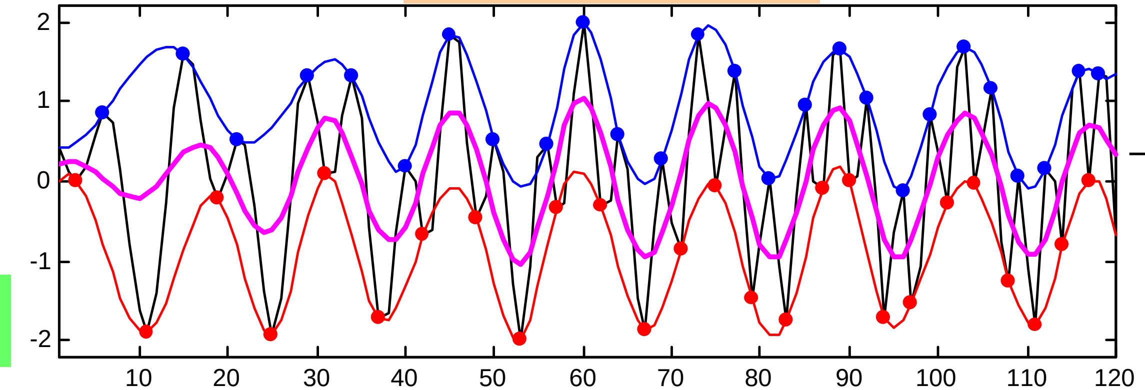
$X(t)$



Step5：由極大值包絡線與極小值包絡線取得均值包絡線

📖 EMD 過程

IMF1, iteration 0

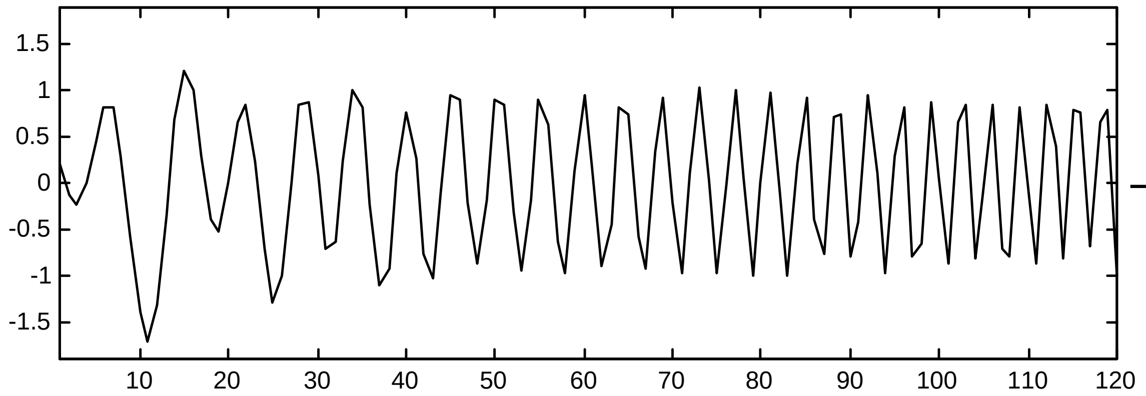


$X(t)$

$X(t) - m_1 = h_1$

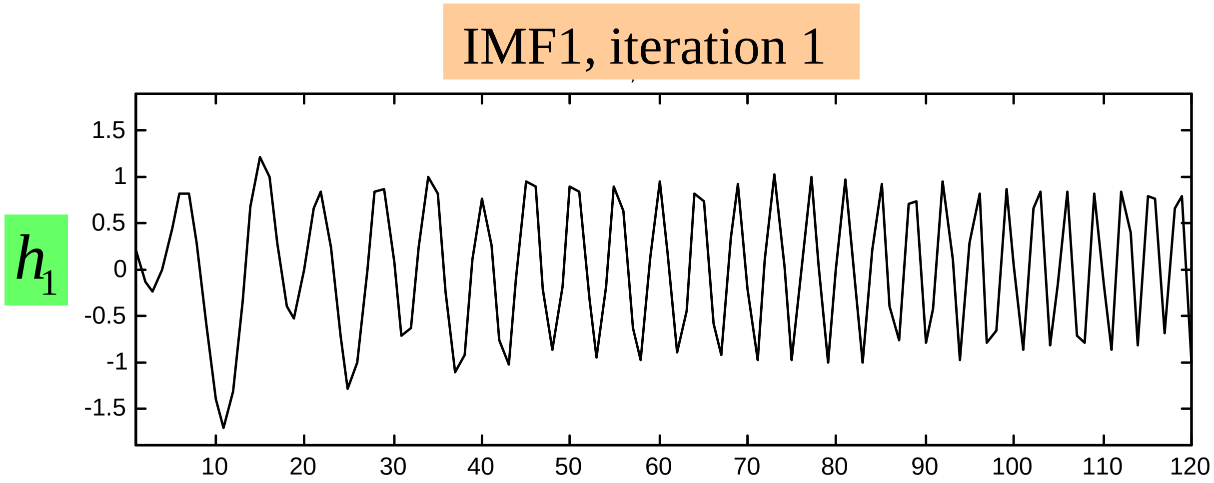
m_1

Step6：原始訊號與均值包絡線之差即是第一個分量 h_1



h_1

📖 EMD 過程



- 篩檢過程有兩個目的

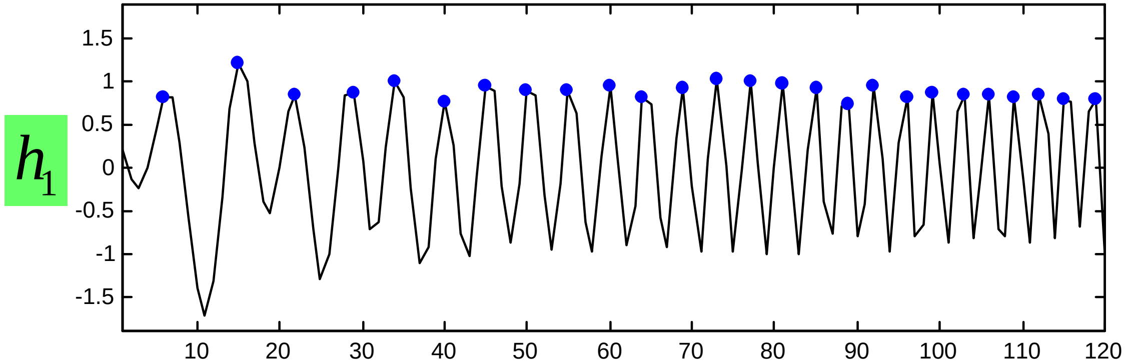
  1. 可消除載波

  2. 使得波形更對稱

  篩檢過程就必須重複進行很多次方能達成這些結果

📖 EMD 過程

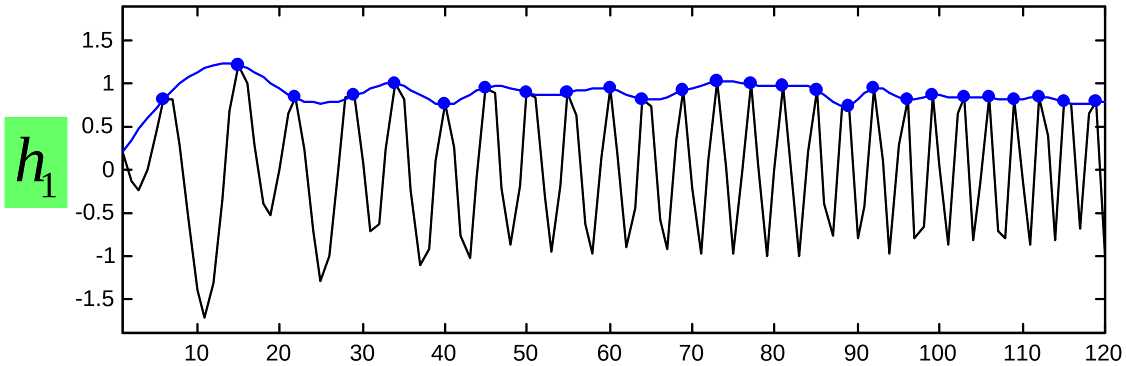
IMF1, iteration 1



Step1：找出局部極大值

🕮 EMD 過程

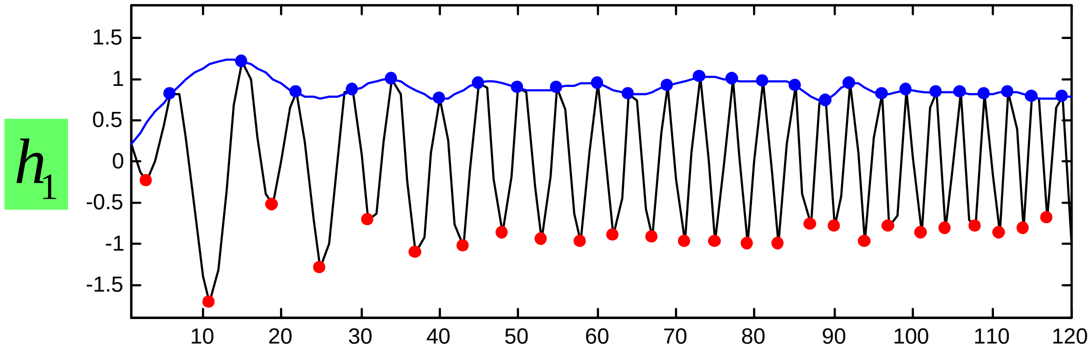
IMF1, iteration 1



Step2：找出局部極大值的包絡線

📖 EMD 過程

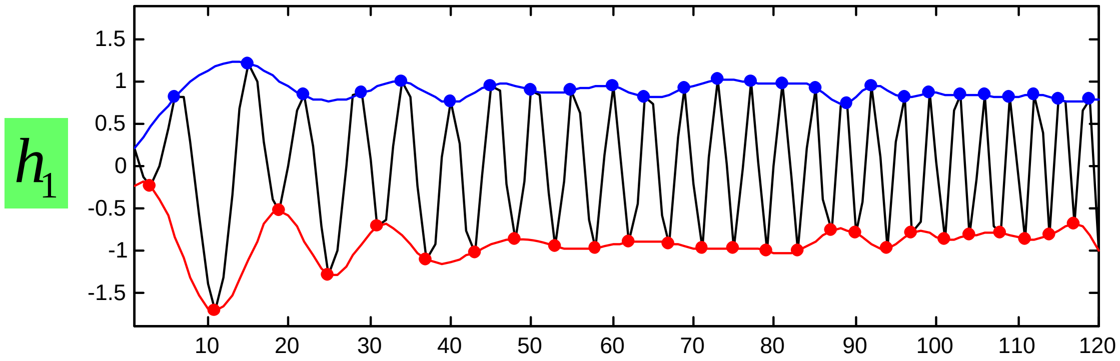
IMF1, iteration 1



Step3：找出局部極小值

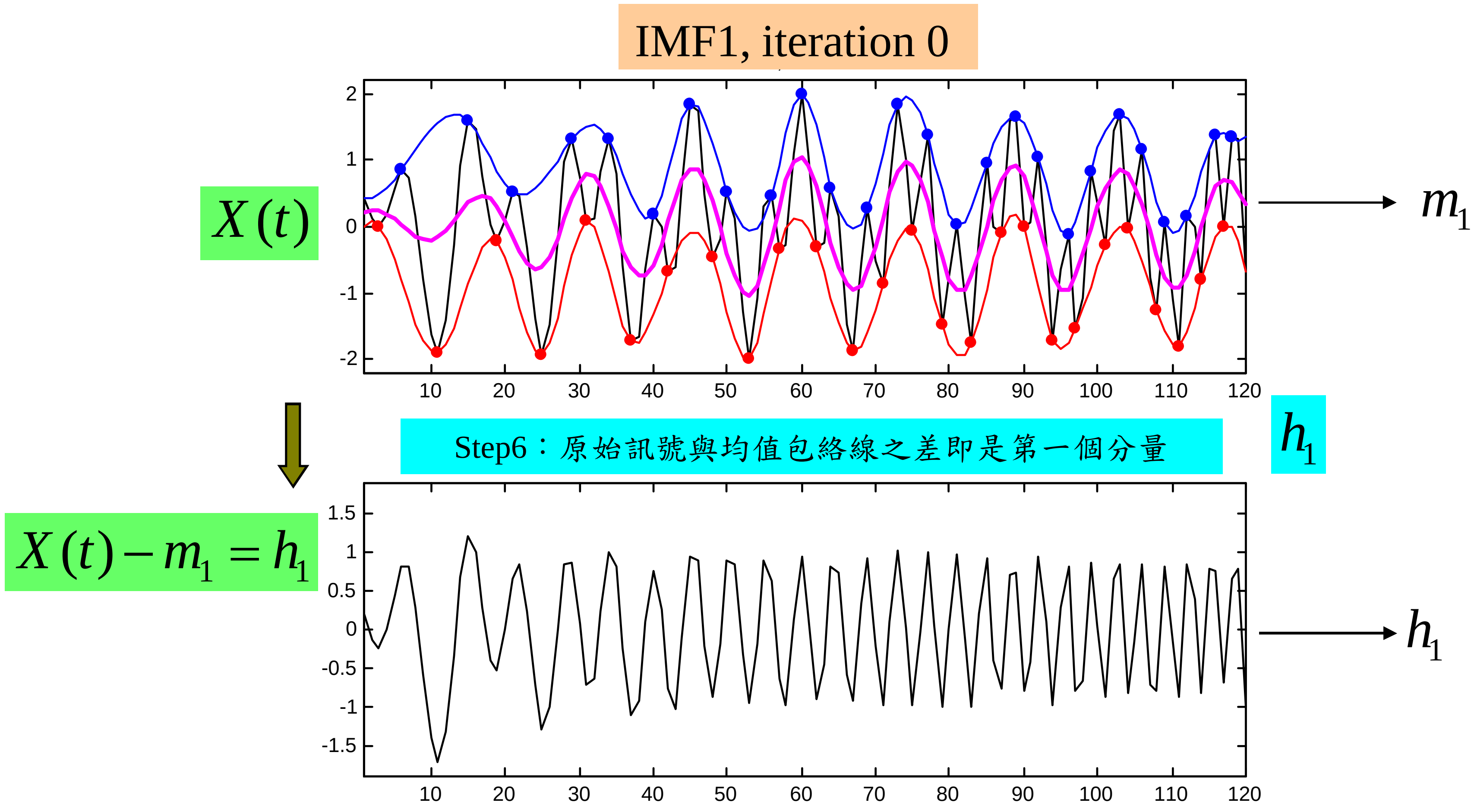
📖 EMD 過程

IMF1, iteration 1



Step4：找出局部極小值的包絡線

📖 EMD 過程

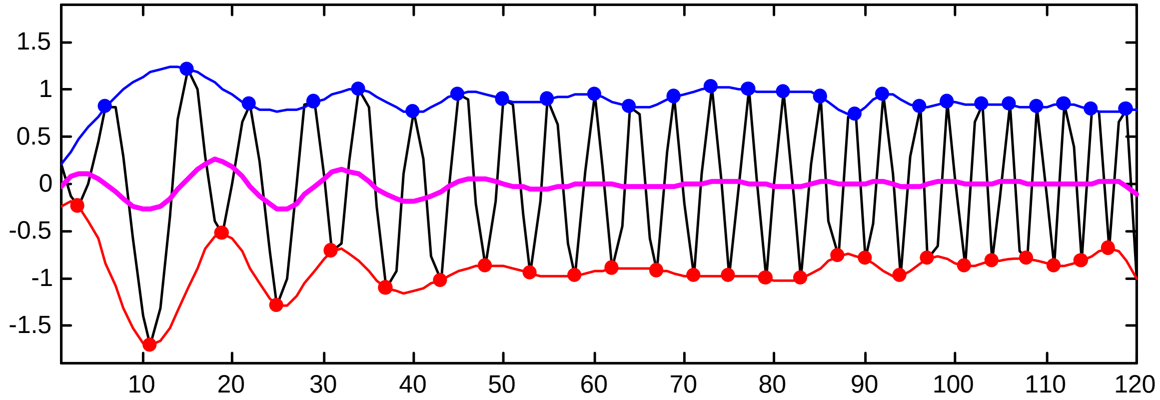


IMF1, iteration 0
X(t)
m1
Step6：原始訊號與均值包絡線之差即是第一個分量
h1
X(t) − m1 = h1
h1

EMD 過程

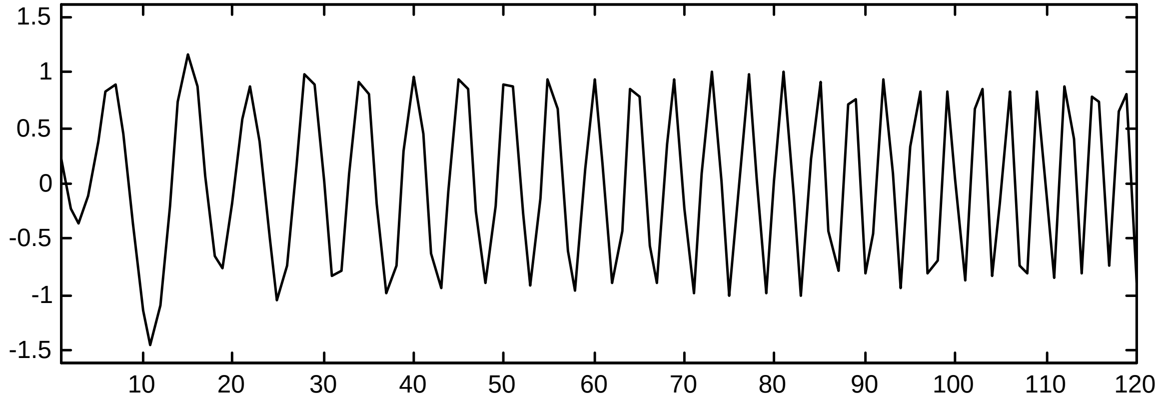
IMF1, iteration 1

h_{11}



residue

$h_1 - m_{11} = h_{11}$

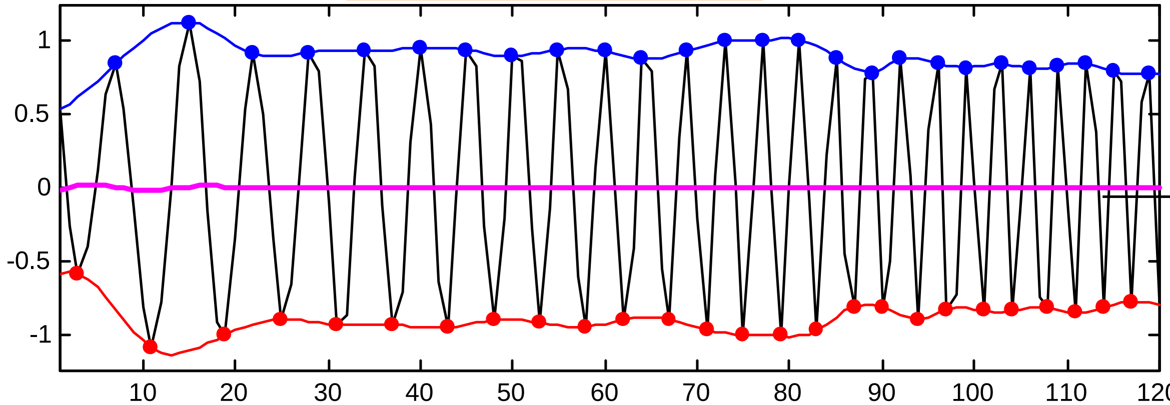


EMD 過程

$h_{1(k-1)}$

IMF1, iteration 8

$$SD = \left\{ \sum_{t=0}^{T} \left[\frac{\left|h_{1(k-1)}(t) - h_{1k}(t)\right|^2}{h^2_{1(k-1)}(t)} \right] \right\}^{1/2}$$

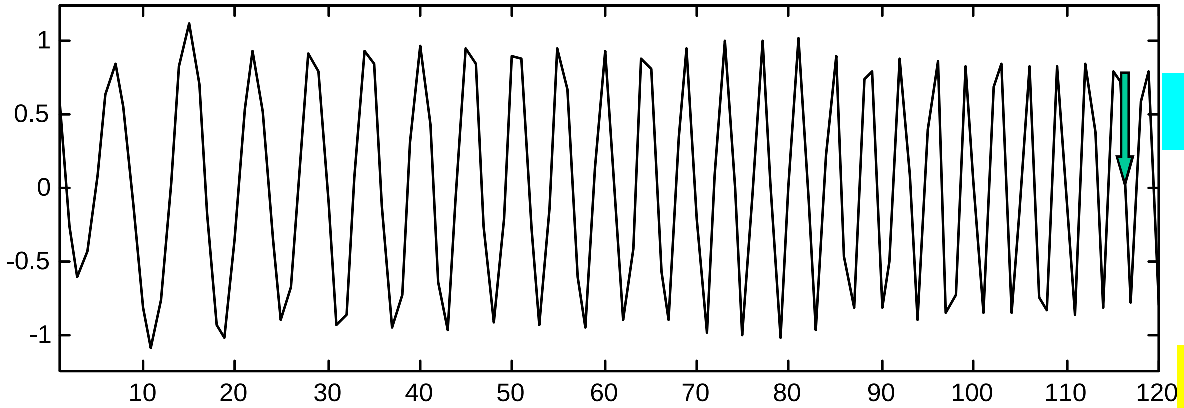


m_{1k}

$h_{1(k-1)} - m_{1k} = h_{1k}$

residue

h_{1k}



SD <0.1

c_1

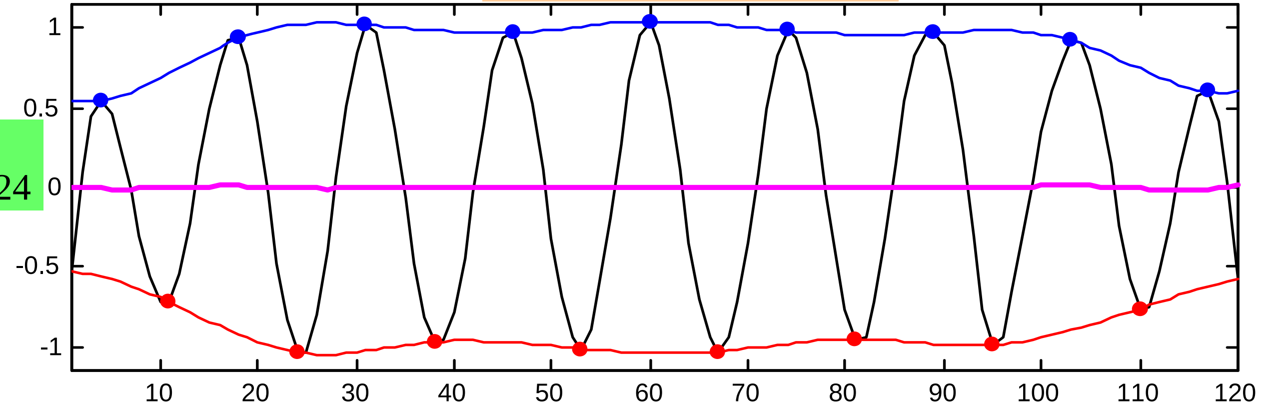
||

IMF1

📖 EMD 過程

IMF2, iteration 5

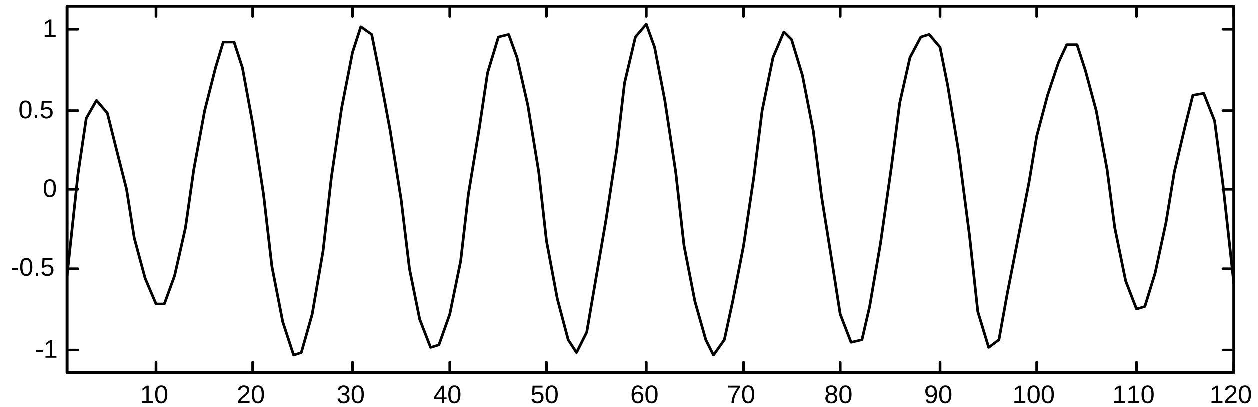
h_{24}



m_{2k}

$$h_{2(k-1)} - m_{2k} = h_{2k}$$

residue



h_{2k}

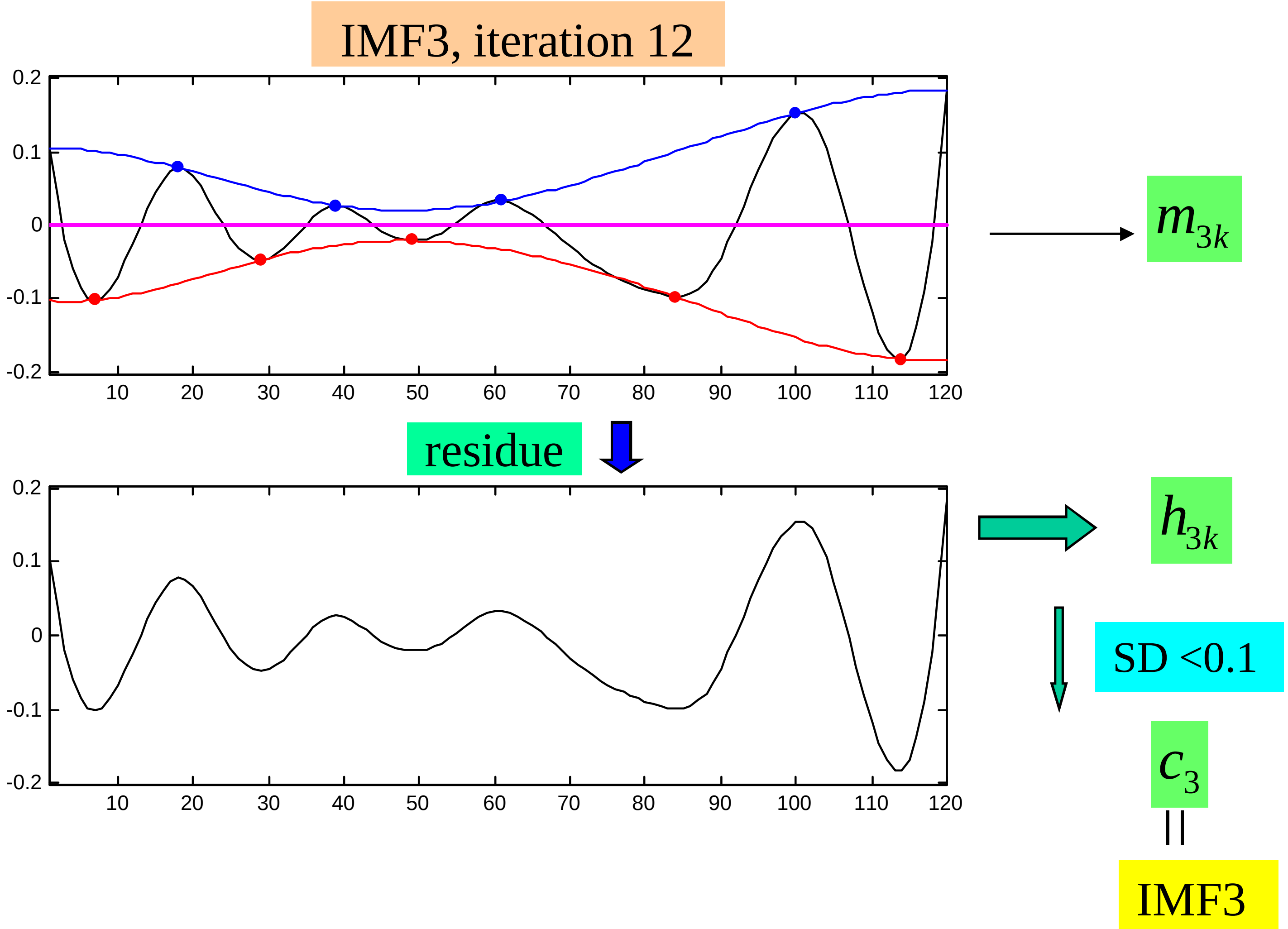
SD <0.1

c_2

||

IMF2

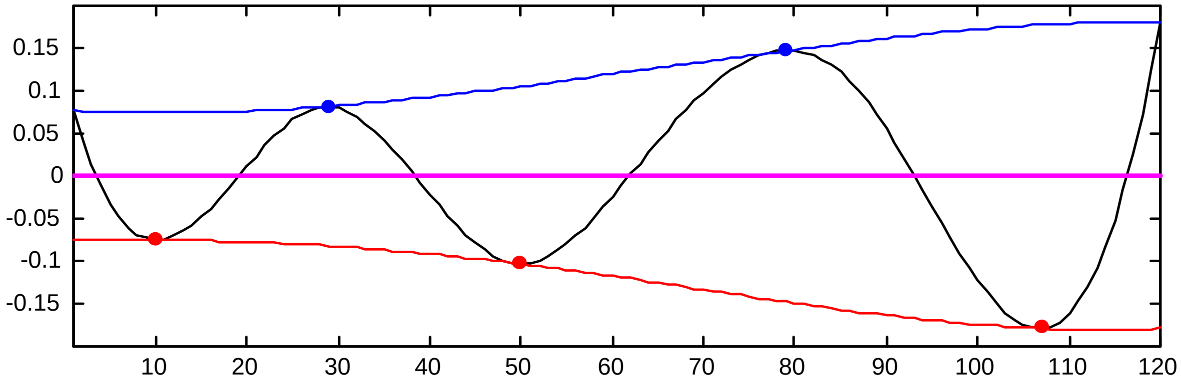
📖 EMD 過程



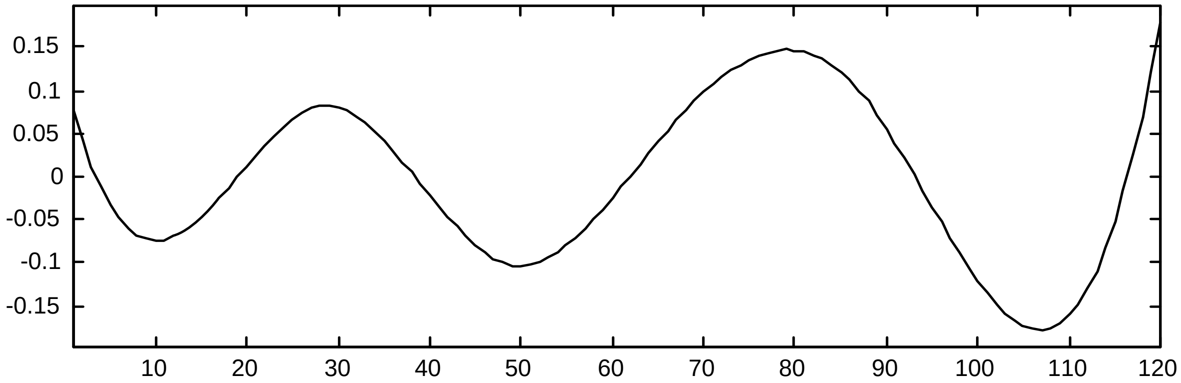
IMF3, iteration 12
0.2
0.1
0
-0.1
-0.2
10 20 30 40 50 60 70 80 90 100 110 120
m_{3k}
residue
h_{3k}
SD <0.1
c_3
||
IMF3

EMD 過程

IMF4, iteration 16

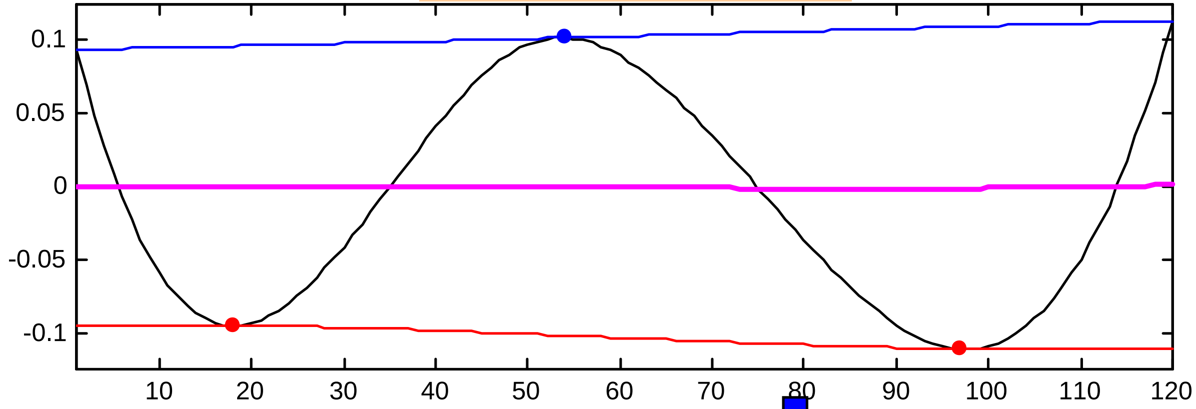


residue

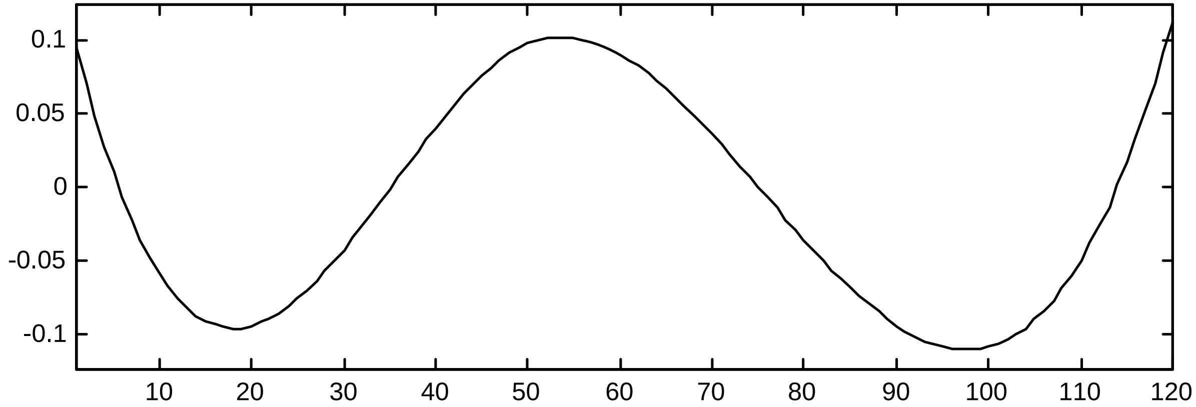


📖 EMD 過程

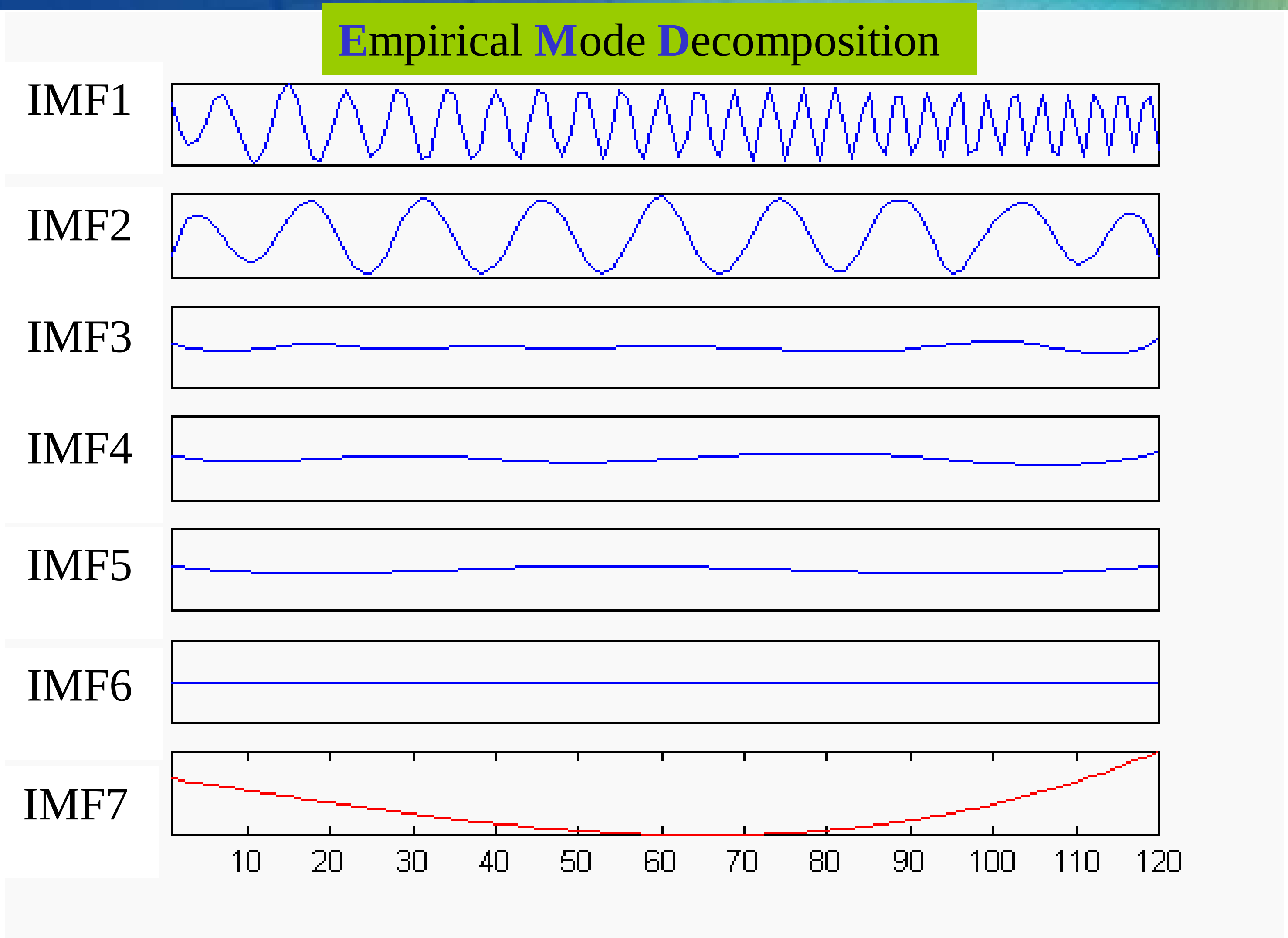
IMF5, iteration 11



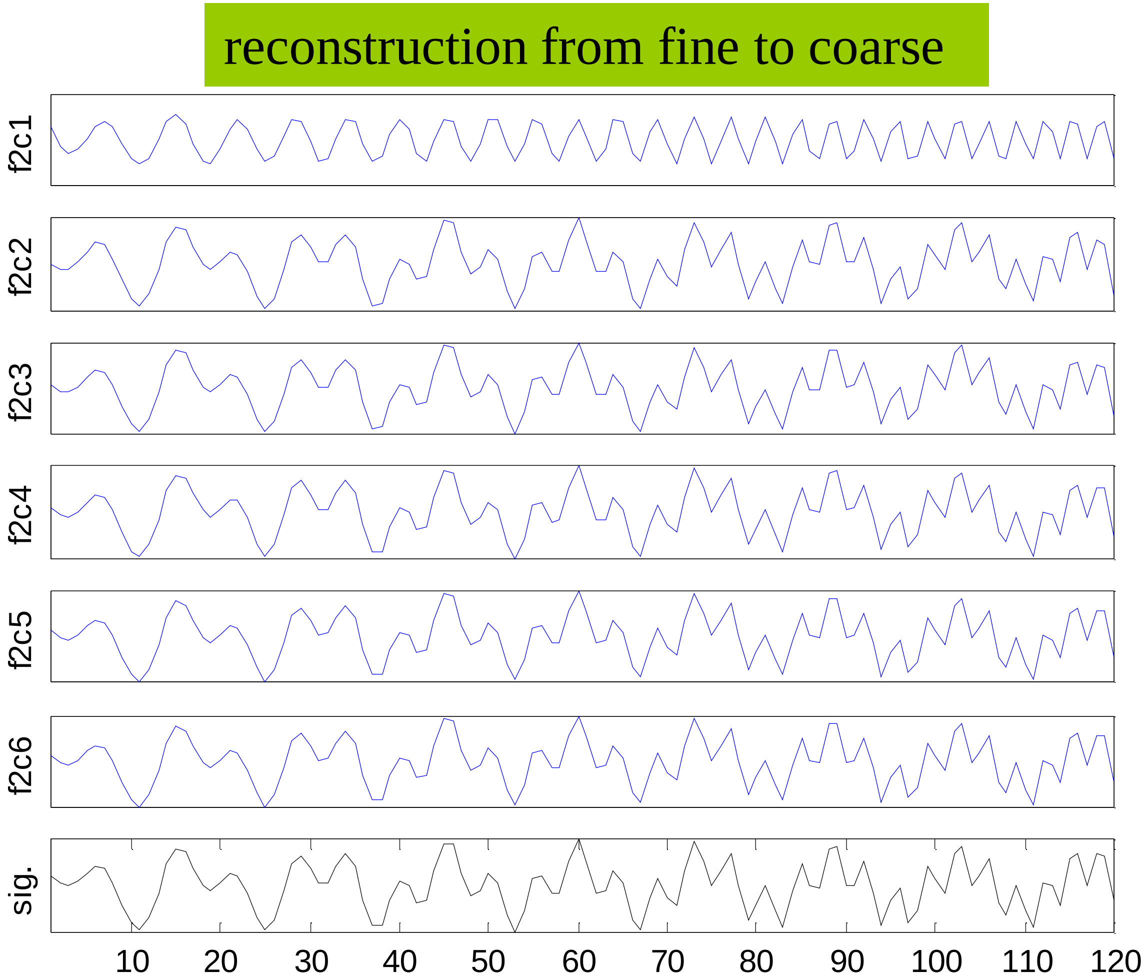
residue



📖 EMD 過程

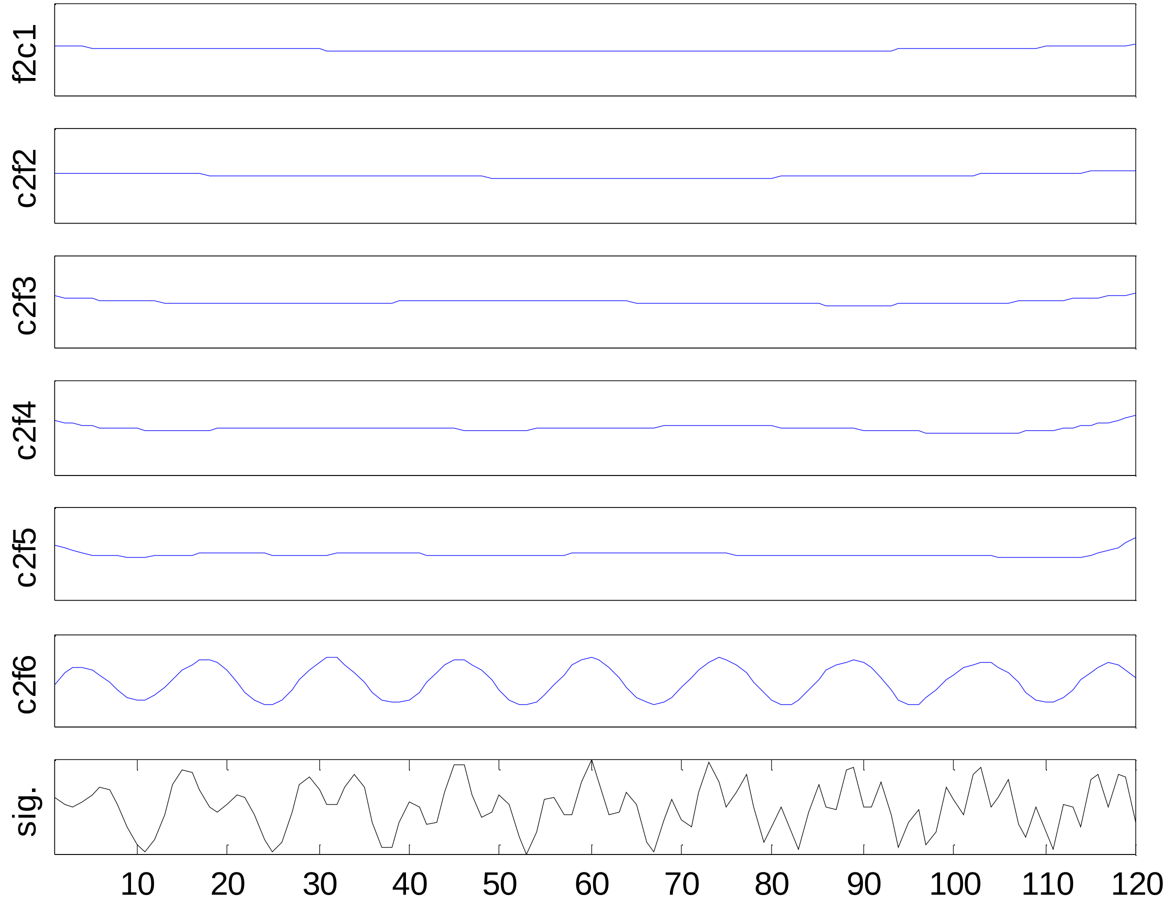

Empirical Mode Decomposition
IMF1
IMF2
IMF3
IMF4
IMF5
IMF6
IMF7
10
20
30
40
50
60
70
80
90
100
110
120

📖 EMD 過程


reconstruction from fine to coarse
f2c1
f2c2
f2c3
f2c4
f2c5
f2c6
sig.
10
20
30
40
50
60
70
80
90
100
110
120

📖 EMD 過程

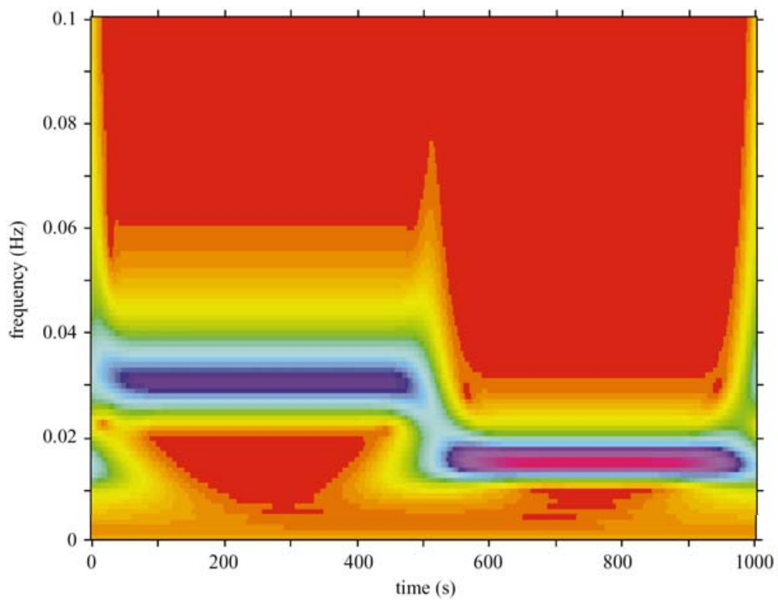
reconstruction from coarse to fine



希爾伯特頻譜 (Hilbert Spectrum)

將原訊號藉由內部模態函數分解 IMF 分量，藉由希爾伯特轉換而得到希爾伯特頻譜。對每一個 IMF 分量做希爾伯特轉換之後，將資料表達成下列的形式

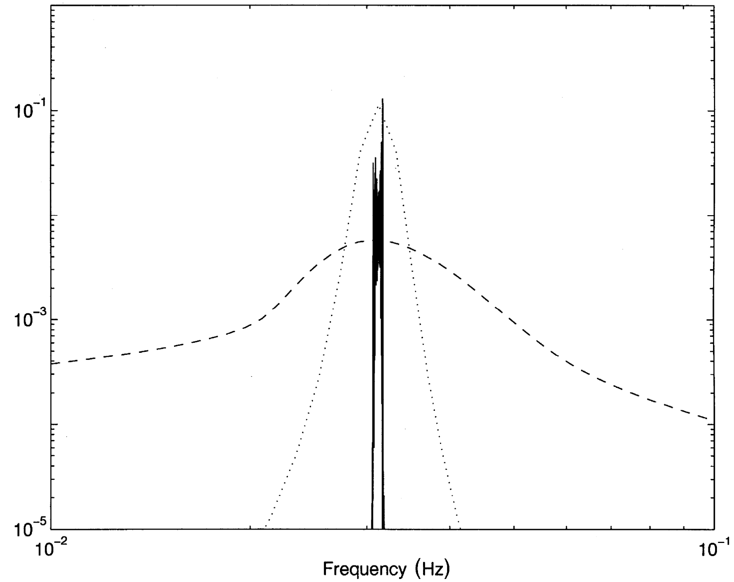
$$X(t) = a_j(t)e^{i2\pi\int f_j(t)dt}$$



📖 二維邊際頻譜 (Marginal Spertrum)

由希爾伯特振幅頻譜對時間積分，可定義邊際頻譜 $h(\omega)$

$$h(\omega) = \int_0^T H(\omega, t)dt$$

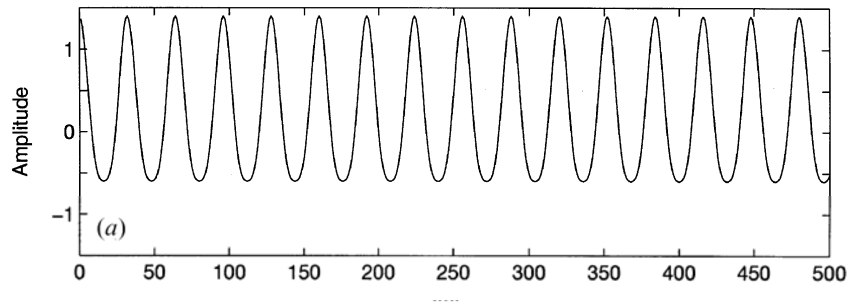


希爾伯特振幅頻譜可視於時間-頻率平面上，涵蓋著由原始訊號高低起伏的振幅或能量所構成的曲面，如對時間積分，則可表示每個頻率所對應到的振幅或能量的總和。因此，邊際頻譜提供了每個頻率的總振幅或總能量的量測。

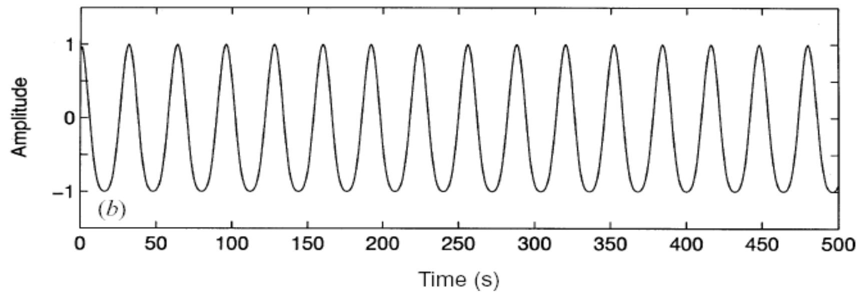
希爾伯特頻譜 (Hilbert Spectrum)

2nd Stoke wave

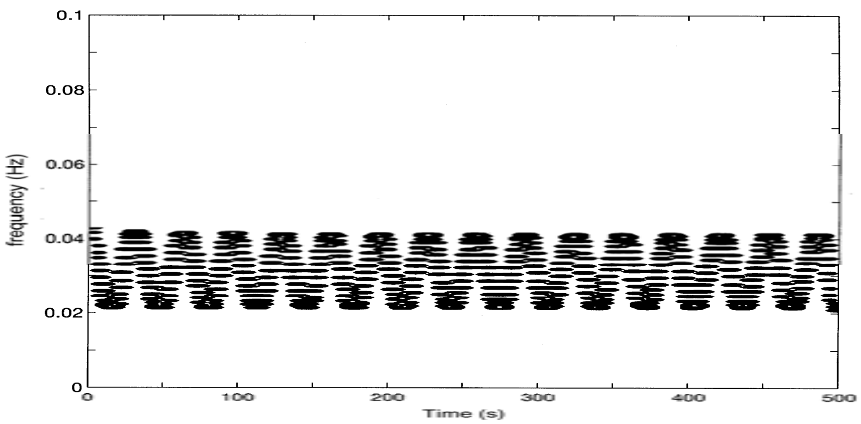
$X(t) = 1.6 + \cos \omega t + 1.6 \cos 2\omega t + \ldots$



IMF



Hilbert Spectrum



📖 希爾伯特頻譜 (Hilbert Spectrum)

$$X(t) = \sin(2\pi t)$$

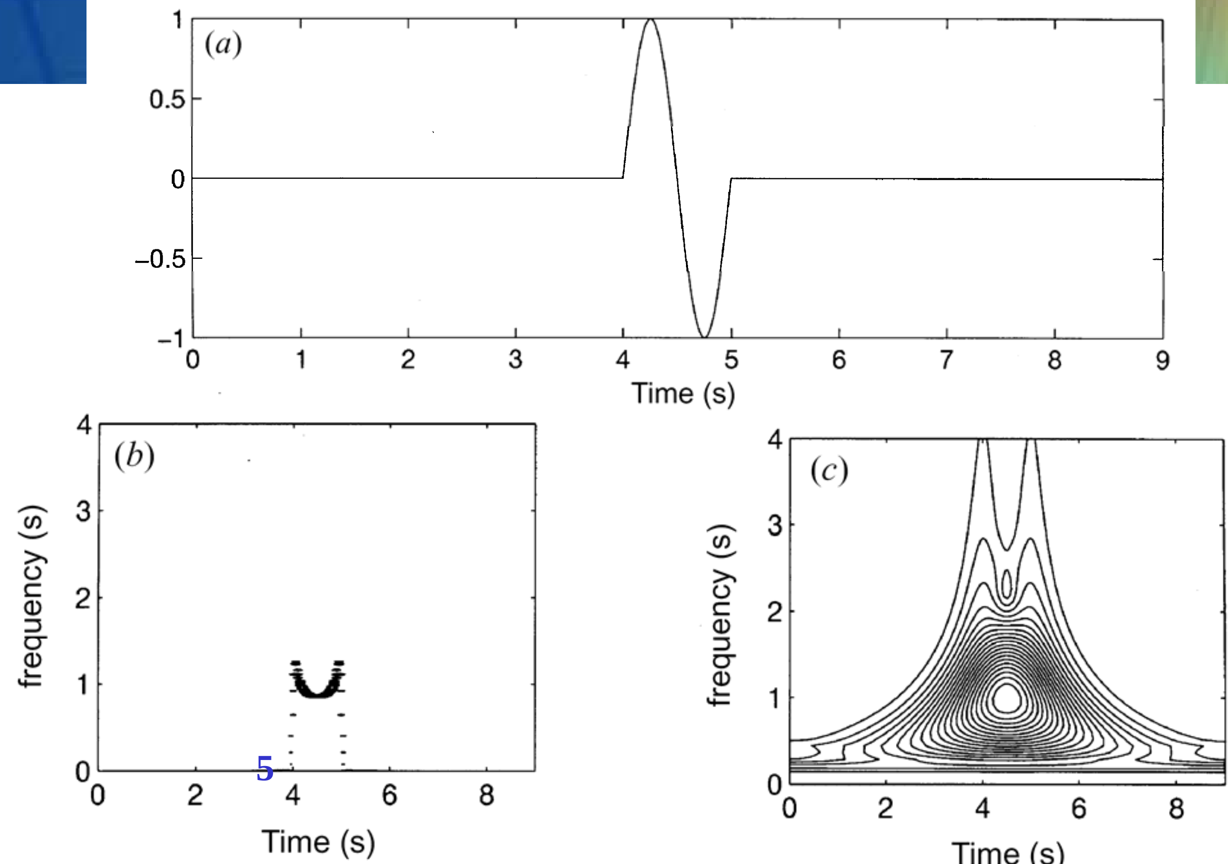
$$t = 4 \sim 5 \text{ sec}$$

$$2\pi f = 2\pi$$

$$\downarrow$$

$$f = 1$$

$$f = 1 \rightarrow$$



A calibration of time localization of the Hilbert spectrum analysis

(a) The calibration data, a single sine wave (b) The Hilbert spectrum for the calibration signal

(c) The Morlet wavelet spectrum for the calibration signal

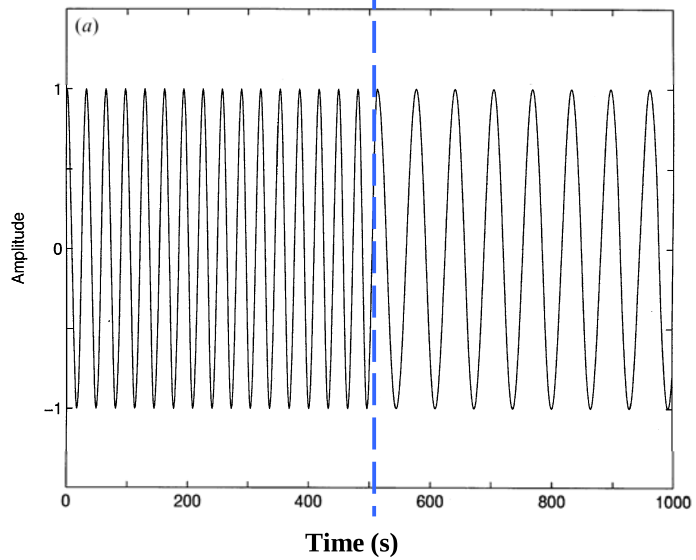
希爾伯特頻譜 (Hilbert Spectrum)

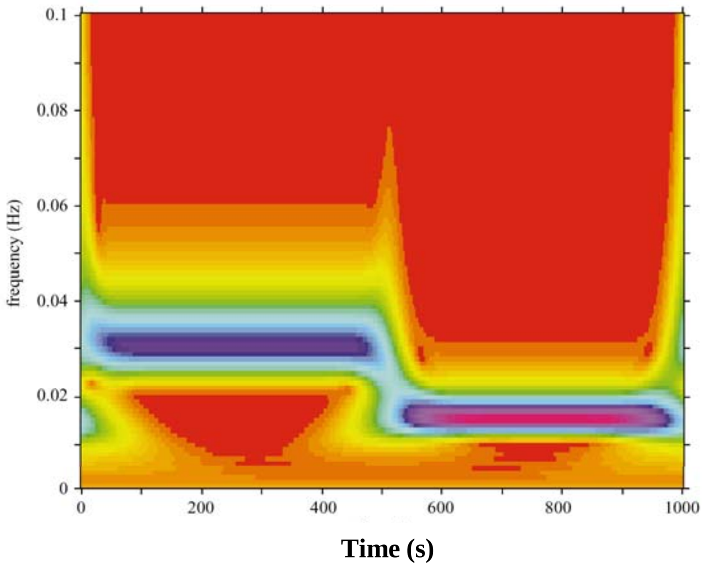
$$X(t)=\cos\left(\frac{1}{16}\pi t\right),\quad t = 1 \sim 512\ \text{s}$$

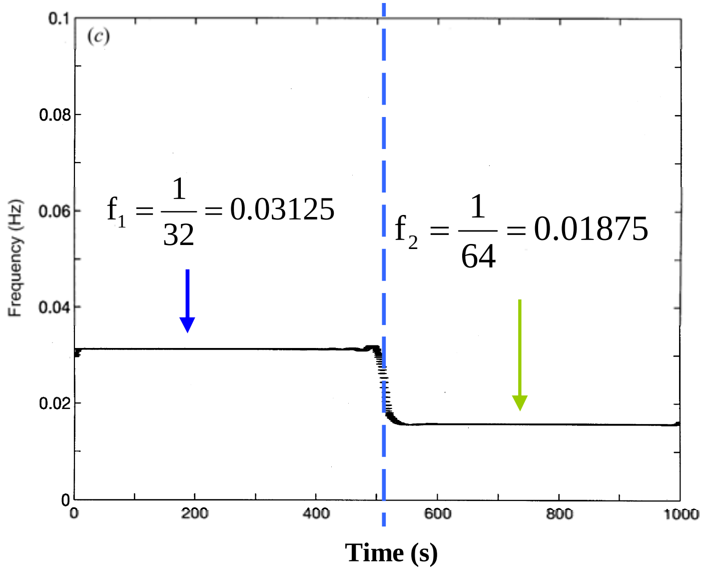
$$X(t)=\cos\left(\frac{1}{32}\pi t\right),\quad t = 513 \sim 1012\ \text{s}$$

$$2\pi f_1=\frac{1}{16}\pi \;\Rightarrow\; f_1=\frac{1}{32}=0.03125$$

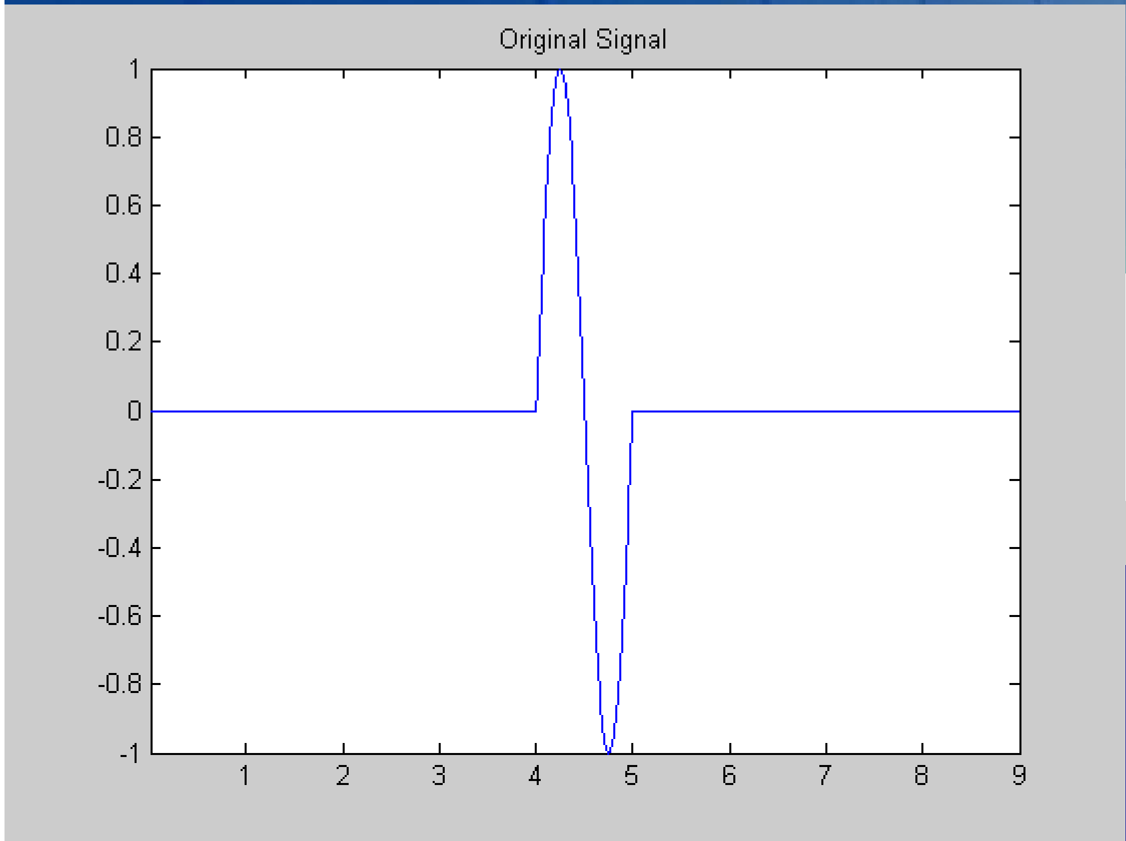
$$2\pi f_2=\frac{1}{32}\pi \;\Rightarrow\; f_2=\frac{1}{64}=0.01875$$







Morlet wavelet spectrum



$$X(t) = \sin(2\pi t)$$

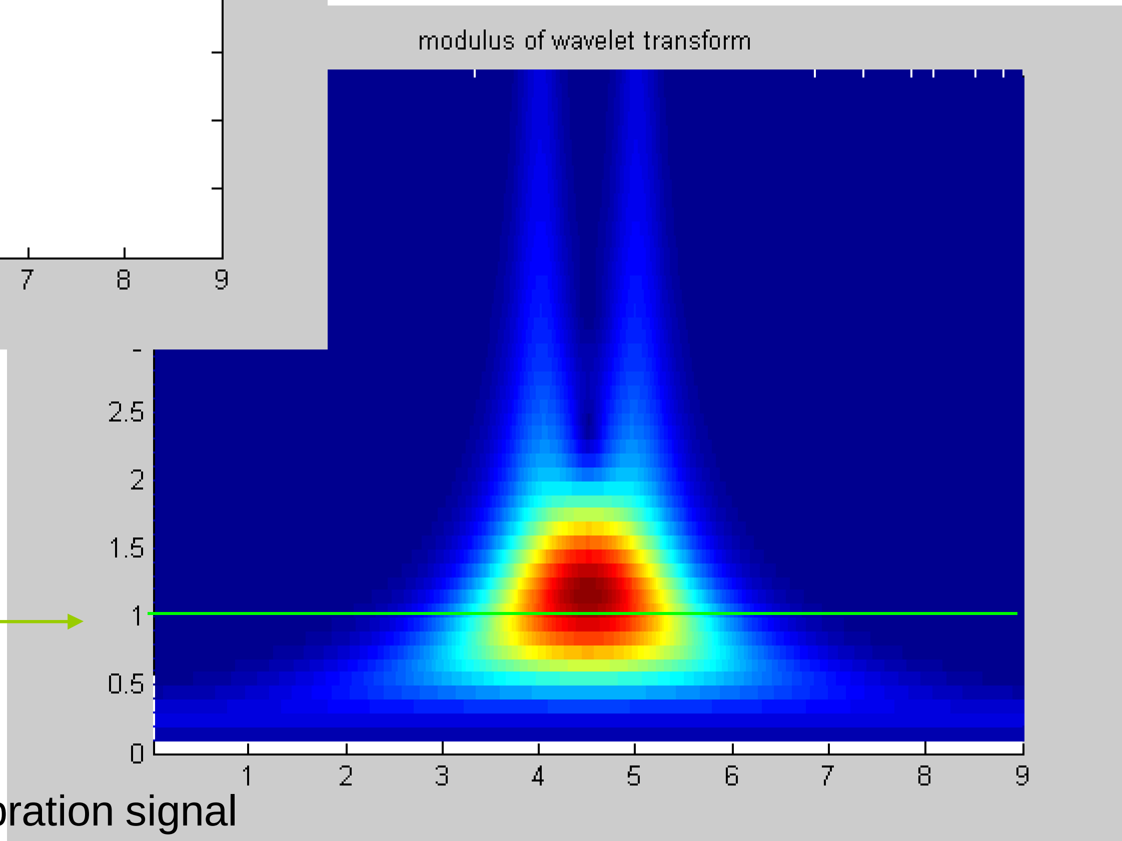
$$t = 4 \sim 5 \text{ sec}$$

$$2\pi f = 2\pi$$

$$\Downarrow$$

$$f = 1$$

$f = 1$



The Morlet wavelet spectrum for the calibration signal